Unit 8, Lesson 9

Using Tables for Conditional Probability

Goals

- Interpret (in written language) two-way tables to estimate probabilities including conditional probability.
- Use probabilities, including conditional probability, to justify (orally and in writing) that events are dependent or independent.

Learning Targets

- I can estimate probabilities, including conditional probabilities, from two-way tables.
- I can use probabilities and conditional probabilities to decide if events are independent.

Lesson Narrative

In this lesson, students create and use two-way tables to estimate conditional probabilities and to decide if events are independent. Two-way tables are useful for keeping track of combinations of events and can be used to estimate probabilities from data.

Technology isn't required for this lesson, but there are opportunities for students to choose to use appropriate technology to solve problems (MP5). We recommend making technology available.

Standards

Building On	**7.NS.A.2**
Addressing	**HSS-CP.A.4**
Building Toward	**HSS-CP.A.4**

Instructional Routines

- Extend It
- Math Talk
- MLR8: Discussion Supports

Student Facing Learning Goals

Let’s use tables to estimate conditional probabilities.

9.1 Math Talk: Fractions in Fractions

Warm-up

5 min

Activity Narrative

This *Math Talk* focuses on dividing fractions. It encourages students to think about methods for dividing fractions and to rely on strategies such as multiplying by an inverse to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students compute probabilities by dividing other probabilities in fractional form.

To divide fractions, students need to look for and make use of structure (MP7).

Standards

Building On	**7.NS.A.2**
Building Toward	**HSS-CP.A.4**

Instructional Routines

- Math Talk
- MLR8: Discussion Supports

Launch

Tell students to close their books or devices (or to keep them closed). Reveal one problem at a time. For each problem:

- Give students quiet think time, and ask them to give a signal when they have an answer and a strategy.
- Invite students to share their strategies, and record and display their responses for all to see.
- Use the questions in the *Activity Synthesis* to involve more students in the conversation before moving to the next problem.

Keep all previous problems and work displayed throughout the talk.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory, Organization

Student Task Statement

Find the value of each expression mentally.

- $\frac{7}{11} \div \frac{8}{11}$
- $\frac{\left(\frac{1}{3}\right)}{\left(\frac{2}{3}\right)}$
- $\frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)}$
- $\frac{\left(\frac{3}{8}\right)\left(\frac{8}{19}\right)}{\left(\frac{5}{19}\right)}$

Student Response

- $\frac{7}{8}$. Sample reasoning: Dividing 7 elevenths by 8 elevenths is the same as dividing 7 by 8 because the denominators are the same.
- $\frac{1}{2}$. Sample reasoning: Multiply the numerator and denominator by 3 to get $\frac{1}{2}$.
- $\frac{2}{4}$ or $\frac{1}{2}$. Sample reasoning: Multiply the expression by $\frac{4}{4}$, which is the same as 1, to get $\frac{1}{2}$.
- $\frac{3}{5}$. Sample reasoning: Multiply the numerator to get $\frac{\left(\frac{3}{19}\right)}{\left(\frac{5}{19}\right)}$. Then multiply by $\frac{19}{19}$ to get $\frac{3}{5}$.

Activity Synthesis

To involve more students in the conversation, consider asking:

- "Who can restate ________'s reasoning in a different way?"
- "Did anyone use the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ________'s strategy?"
- "Do you agree or disagree? Why?"
- "What connections to previous problems do you see?"

Access for English Language Learners

MLR8 Discussion Supports. Display sentence frames to support students when they explain their strategy. Examples:

- "First, I ________ because ________."
- "I noticed ________, so I ________."

Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking, Representing

9.2 A Possible Cure

15 min

Activity Narrative

In this activity, students use two-way tables to represent the outcomes in a sample space. Turning the two-way table into a relative frequency table shows estimates for probabilities, which can then be used to decide if events are independent.

Making spreadsheet technology available gives students an opportunity to choose appropriate tools strategically (MP5).

Standards

Addressing **HSS-CP.A.4**

Instructional Routines

- Extend It
- MLR8: Discussion Supports

Launch

Give students 10 minutes to work on the problems, and then pause for a brief whole-class discussion.

Access for Students with Disabilities

Action and Expression: Provide Access for Physical Action. Support effective and efficient use of tools and assistive technologies. To use spreadsheet software, some students may benefit from a checklist or a list of steps to be

able to use the technology.
Supports accessibility for: Organization, Memory, Attention

Student Task Statement

A pharmaceutical company is testing a new medicine for a disease, using 115 test subjects. Some of the test subjects are given the new medicine and others are given a placebo. The results of their tests are summarized in the table.

	no more symptoms	symptoms persist	total
given medicine	31	26	57
given placebo	16	42	58
total	47	68	115

1. Divide the value in each cell by the total number of test subjects to find each probability to two decimal places. Some of the values have been completed for you.

	no more symptoms	symptoms persist	total
given medicine	0.27		0.50
given placebo			
total			1

If one of these test subjects is selected at random, find each probability:

a. P(symptoms persist)

b. P(given medicine and symptoms persist)

c. P(given placebo or symptoms persist)

2. From the original table, divide each cell by the total for the row to find the probabilities with row conditions. Some of the values have been completed for you.

	no more symptoms	symptoms persist	total
given medicine	0.54		
given placebo			1

a. $P(\text{symptoms persist} \mid \text{given medicine})$

b. $P(\text{no more symptoms} \mid \text{given placebo})$

3. Jada didn't read the instructions for the previous problem well and used the table she created on the first problem to divide each cell by the probability total for each row. For example, in the top left cell she calculated $0.27 \div 0.5$. Complete the table using Jada's method.

	no more symptoms	symptoms persist	total
given medicine			
given placebo			

What do you notice about this table?

4. From the original table, divide each cell by the total for the column to find the probabilities with column conditions. Some of the values have been completed for you.

	no more symptoms	symptoms persist
given medicine	0.66	
given placebo		
total		1

   a. $P(\text{given medicine} \mid \text{symptoms persist})$

   b. $P(\text{given placebo} \mid \text{no more symptoms})$

5. Are the events "symptoms persist" and "given medicine" independent events? Explain or show your reasoning.

6. Based on your work, does being given this medicine have an affect on whether symptoms persist or not?

Student Response

1.

	no more symptoms	symptoms persist	total
given medicine	0.27	0.23	0.50
given placebo	0.14	0.37	0.50
total	0.41	0.59	1

   a. 0.60 or 0.59 depending on rounding

   b. 0.23

   c. 0.74 or 0.73 depending on rounding

2.

	no more symptoms	symptoms persist	total
given medicine	0.54	0.46	1
given placebo	0.28	0.72	1

   a. 0.46

   b. 0.28

3. Sample response: Jada's table has almost all of the same values as the table I filled in using the right instructions. Only the value for the subjects who were "given placebo and have symptoms persist" is off of the other value by 0.02, which may be due to rounding errors.

4.

	no more symptoms	symptoms persist
given medicine	0.66	0.38
given placebo	0.34	0.62
total	1	1

a. 0.38

b. 0.34

5. They are not independent events. Sample reasoning: $P(\text{symptoms persist}) = 0.6$ and $P(\text{symptoms persist} \mid \text{given medicine}) = 0.46$. If the events were independent, these probabilities would be equal.

6. Sample response: Yes. Because the events are dependent and the probability of symptoms persisting after being given medicine is much less than the probability of symptoms persisting overall, I think the medicine has an effect on whether the symptoms persist.

Are You Ready for More?

Consider the data collected from all students in grades 11 and 12 and their intentions of going to prom.

1. 65% of the students who are going to prom are from grade 12. Complete the table.

	going to prom	not going to prom	total
grade 11			127
grade 12			116
total	124	119	243

2. Based on your results, what is the probability that a student from this group, selected at random, is a student in grade 11 that did not go to prom?

Extension Student Response

1.

	going to prom	not going to prom	total
grade 11	43	84	127
grade 12	81	35	116
total	124	119	243

2. $\frac{84}{243}$

Activity Synthesis

The purpose of this discussion is for students to interpret probabilities in a two-way table.

Ask students to interpret each of the probabilities they found. For example, "What does the probability $P(\text{symptoms persist} \mid \text{given medicine})$ mean in the context of the problem?"

Here are sample interpretations of the probabilities.

- $P(\text{symptoms persist})$: The probability that a test subject chosen at random from all of the subjects still had symptoms after the test.
- $P(\text{given medicine and symptoms persist})$: The probability that a test subject chosen at random from all of the subjects was among those given medicine and still had symptoms after the test.
- $P(\text{given placebo or symptoms persist})$: The probability that a test subject chosen at random from all of the subjects still had symptoms after the test or was given a placebo.
- $P(\text{symptoms persist} \mid \text{given medicine})$: The probability that a test subject chosen at random from those who were given medicine had symptoms after the test.
- $P(\text{no more symptoms} \mid \text{given placebo})$: The probability that a test subject chosen at random from those who were given placebo had no more symptoms after the test
- $P(\text{given medicine} \mid \text{symptoms persist})$: The probability that a test subject chosen at random from those who had symptoms after the test was given medicine.
- $P(\text{given placebo} \mid \text{no more symptoms})$: The probability that a test subject chosen at random from those who had no more symptoms after the test was given placebo.

Display what Jada did when creating the top left cell in table:

$$\frac{P(\text{no more symptoms and given medicine})}{P(\text{given medicine})}$$

Here are some questions for discussion.

- "What do you notice about what Jada did and $P(\text{no more symptoms} \mid \text{given medicine})$?" (I noticed that they are equal.)
- "What does the equality look like when applied to general events A and B." ($P(\text{A} \mid \text{B}) = \frac{P(\text{A and B})}{P(\text{B})}$)
- "What is the relationship between this equation and the multiplication rule?" (From the multiplication rule, we know that $P(\text{A and B}) = P(\text{A} \mid \text{B}) \cdot P(\text{B})$. If we divide both sides by $P(\text{B})$, we get the equation that Jada used.)
- "How did you determine whether the events were dependent or independent?" (I determined that they were dependent events because $P(\text{symptoms persist}) = 0.6$ and $P(\text{symptoms persist} \mid \text{given medicine}) = 0.46$ were not equal. If the events were independent, these probabilities would be equal.)

Access for English Language Learners

MLR8 Discussion Supports. For each observation that is shared, invite students to turn to a partner and restate what they heard, using precise mathematical language.
Advances: Listening, Speaking

9.3 The Blood Bank

10 min

Activity Narrative

In this activity, students use a two-way table to estimate probabilities, including conditional and compound events.

Standards

Addressing **HSS-CP.A.4**

Launch

Arrange students in groups of 2. Give students quiet time to work on the questions, have partners compare answers, and then have a whole-class discussion.

Access for Students with Disabilities

Representation: Access for Perception. Provide appropriate reading accommodations and supports to ensure student access to written directions, word problems, and other text-based content.
Supports accessibility for: Language

Student Task Statement

A blood bank in a region has some information about the blood types of people in its community. Blood is grouped into types O, A, B, and AB. Each blood type either has the Rh factor (Rh+) or not (Rh-). If a person is randomly selected from the community, the probability of that person having each blood type and Rh factor combination is shown in the table.

	O	A	B	AB	total
Rh+	0.374	0.357	0.085	0.034	
Rh-	0.066	0.063	0.015	0.006	
total					1

1. What does the 0.085 in the table represent?
2. Use the table or create additional tables to find the probabilities, then describe the meaning of the event.
    a. $P(\text{O})$
    b. $P(\text{Rh+})$
    c. $P(\text{O and Rh+})$
    d. $P(\text{O or Rh+})$
    e. $P(\text{O} \mid \text{Rh+})$
    f. $P(\text{Rh+} \mid \text{O})$

Student Response

1. The probability that the randomly selected person from the community has type B blood and the Rh factor.
2. a. 0.44 is the probability of the person selected at random having type O blood.
   b. 0.85 is the probability of the person selected at random having the Rh factor.
   c. 0.374 is the probability of the person selected at random having type O blood and the Rh factor.
   d. 0.916 is the probability of the person selected at random having type O blood or the Rh factor.
   e. 0.44 is the probability of the person having type O blood when selected at random from among those who have the Rh factor.
   f. 0.85 is the probability of the person having the Rh factor when selected at random from among those who have type O blood.

Building on Student Thinking

Students might struggle with using decimal values in the two-way table. Ask students how many people would be in each combination of groups if there are 1,000 people in the community.

Activity Synthesis

The goal of this discussion is for students to understand how to estimate conditional probabilities using a two-way table and to informally assess student understanding of notation used with conditional probability.

Here are some questions for discussion.

- "What is the probability that a person selected is Rh- under the condition that they are blood type B?" ($\frac{0.015}{0.1} = 0.15$)
- "How would you write 'the probability that a person selected is Rh- under the condition that they are blood type B' using probability notation?" ($P(\text{Rh- } | \text{ B})$)
- "What is the probability that a person selected has blood type AB under the condition that they are Rh-?" ($\frac{0.006}{0.15} = 0.04$)
- "A person who has blood type A and is Rh+ can give blood to anyone who has either type A or AB blood and is Rh+. What is the probability that someone with type A blood that is Rh+ can be used for the next person needing blood?" (0.391 because $0.357 + 0.034 = 0.391$)

Lesson Synthesis

Display the table from the *Cool-down* and its description.

A large aquarium has 48 guppy fish in it. The table summarizes the fish based on color and sex.

	orange	blue
male	5	13
female	12	18

Here are some questions for discussion.

- “What is the probability that a fish picked at random is orange?” ($\frac{17}{48}$)
- “What is the probability that a fish picked at random is orange under the condition that the fish is female? ($\frac{12}{30}$ or equivalent)
- “What is the probability that a fish picked at random is orange and male?” ($\frac{5}{48}$)
- “What is the probability that a fish picked at random is orange or male?” ($\frac{30}{48}$ or equivalent)
- “What is the probability that a female fish picked at random is blue?” ($\frac{18}{30}$ or equivalent)
- “How would you write ‘what is the probability that a female fish picked at random is blue?’ using probability notation?”($P(\text{blue} \mid \text{female})$)
- “Are the events of being an orange fish and being a female fish independent events?” (They are not independent events. $P(\text{being an orange fish}) = \frac{17}{48}$ and $P(\text{being an orange fish} \mid \text{being a female fish}) = \frac{12}{30}$. If the events were independent, these probabilities would be equal.)

9.4 Guppies

Cool-down

5 min

Standards

Addressing **HSS-CP.A.4**

Student Task Statement

A large aquarium has 48 guppy fish in it. The table summarizes the fish based on color and sex.

	orange	blue
male	5	13
female	12	18

1. If a guppy is selected at random from the aquarium, what is the probability that it is blue and female?
2. If a guppy is selected at random from the aquarium, what is the probability that it is blue?
3. Find the conditional probability $P(\text{female} \mid \text{blue})$. Explain or show your reasoning.

Student Response

1. 0.38
2. 0.65
3. 0.58 because $P(\text{female} \mid \text{blue}) = \frac{P(\text{female and blue})}{P(\text{blue})}$ and $\frac{0.38}{0.65} \approx 0.58$.

Responding to Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas and language addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 9 Summary

Organizing data in tables is a useful way to see information and compute probabilities. Consider the data collected from all students in grades 11 and 12 and their intentions of going to prom.

	going to prom	not going to prom	total
grade 11	43	84	127
grade 12	81	35	116
total	124	119	243

A student is randomly selected from this group of students. We can find a number of probabilities based on the table.

$P(\text{grade 12 and going to prom}) = \frac{81}{243}$ because there are 81 students who are both in 12th grade and going to prom out of all 243 students in the group who could be selected.

$P(\text{grade 12}) = \frac{116}{243}$ because there are 116 grade 12 students out of the entire group.

$P(\text{going to prom}) = \frac{124}{243}$ because there are 124 students going to prom out of the entire group.

$P(\text{grade 12 or going to prom}) = \frac{159}{243}$ since there are 159 students in grade 12 or going to prom (43 from grade 11 going to prom, 81 from grade 12 going to prom, and 35 from grade 12 not going to prom).

$P(\text{going to prom} \mid \text{grade 12}) = \frac{81}{116}$ represents the probability that the chosen student is going to prom under the condition that the student is in grade 12. The group we are considering is different for this probability because the condition is that the student is in grade 12. Imagine all grade 12 students are in a room and we are selecting from only this group. We see that 81 students from this group are going to prom out of the 116 students in the group.

$P(\text{grade 11} \mid \text{going to prom}) = \frac{43}{124}$ represents the probability that the chosen student is in grade 11 under the condition that the student is going to prom. In this case, we imagine that everyone going to prom is gathered in a room, and we are selecting one student from this group. The probability that this student is in grade 11 when considering only this group is $\frac{43}{124}$.

The last two probabilities can also be found using the multiplication rule.

$P(\text{A and B}) = P(\text{A} \mid \text{B}) \cdot P(\text{B})$ can be rewritten $P(\text{A} \mid \text{B}) = \frac{P(\text{A and B})}{P(\text{B})}$.

For example, substituting values into $P(\text{going to prom} \mid \text{grade 12}) = \frac{P(\text{going to prom and grade 12})}{P(\text{grade 12})}$ gives $\frac{81}{116} = \frac{\left(\frac{81}{243}\right)}{\left(\frac{116}{243}\right)}$ which is a true equation.

Lesson 9 Practice Problems

1 Student Task Statement

A tour company makes trips to see dolphins in the morning and in the afternoon. The two-way table summarizes whether or not customers saw dolphins on a total of 100 different trips.

	morning	afternoon
dolphins	39	34
no dolphins	5	22

a. If a trip is selected at random, what is the probability that customers did not see dolphins on that trip?

b. If a trip is selected at random, what is the probability that customers did not see dolphins under the condition that the trip was in the morning?

c. Are the events of seeing dolphins and the time of the trip (morning or afternoon) dependent or independent events? Explain your reasoning.

Solution

a. $\frac{27}{100}$, 0.27 (or equivalent)

b. $\frac{5}{44}$, 0.114 (or equivalent)

c. Sample response: The events are likely dependent. The probability of not seeing dolphins is about 0.27 ($\frac{27}{100}$), but the probability of not seeing dolphins in the morning is about 0.114 ($\frac{5}{44}$). Because these values are not very close, the events are probably dependent.

2 Student Task Statement

Noah is unsure whether the coin and number cube he has are fair. He flips the coin then rolls the number cube and records the result. He does this a total of 50 times. The results are summarized in the table.

	one	two	three	four	five	six
heads	5	3	5	3	10	1
tails	3	4	5	2	7	2

a. Create a two-way table that displays the probability for each outcome based on Noah's tests.

b. If one of Noah's 50 results is selected at random, what is the probability that the coin was heads?

c. If one of Noah's 50 results is selected at random, what is the probability that the number cube was 5?

d. Do you think the coin is fair? Do you think the cube is fair? Explain your reasoning.

Solution

a.

	one	two	three	four	five	six
heads	0.1	0.06	0.1	0.06	0.2	0.02
tails	0.06	0.08	0.1	0.04	0.14	0.04

b. 0.54

c. 0.34

d. Sample response: The coin seems to be fair, but the number cube is not. I expect the coin to be heads with a probability of 0.5, and it is 0.54, which is fairly close with this number of flips. I expect 5 to be rolled on the number cube with a probability of about 0.17 ($\frac{1}{6}=0.17$), but this cube got almost twice that.

3

Student Task Statement

A student surveys 30 people as part of a project for a statistics class. Here are the survey questions.

- Are you left-handed or right-handed?
- Are you left-eye dominant or right-eye dominant?

The results of the survey are summarized in the two-way table.

	right-eye dominant	left-eye dominant
right-handed	14	11
left-handed	3	2

What is the probability that a person from the survey chosen at random is right-handed under the condition that they are right-eye dominant?

A. $\frac{25}{30}$

B. $\frac{14}{30}$

C. $\frac{14}{25}$

D. $\frac{14}{17}$

Solution

D

4 from Unit 8, Lesson 8

Student Task Statement

Priya flips a fair coin and then rolls a standard number cube. What is the probability that she rolled a

3 under the condition that she flipped heads?

A. $\frac{1}{2}$

B. $\frac{1}{6}$

C. $\frac{1}{12}$

D. $\frac{3}{12}$

Solution

B

5 from Unit 8, Lesson 8

Student Task Statement

Andre flips one fair coin and then flips another fair coin.

a. What is the probability that he gets heads on both coins?
b. What is the probability that he gets heads on the second coin under the condition that the first flip is heads?
c. What is the probability that the second flip is not heads?
d. What is the probability that the first flip is heads and the second flip is not heads?

Solution

a. 0.25
b. 0.5
c. 0.5
d. 0.25

6 from Unit 8, Lesson 7

Student Task Statement

Han randomly selects a card from a standard deck of cards. He places it on his desk and then Jada randomly selects a card from the remaining cards in the same deck.

a. What is the probability that Han selects a card that has diamonds on it?
b. What is the probability that Jada selects a card that has diamonds on it?
c. What is the probability that Han selects a card that has diamonds on it and that Jada selects a card that has diamonds on it?

d. Are the events of Han and Jada randomly selecting a card dependent or independent? Explain your reasoning.

Solution

a. $\frac{1}{4}$

b. It depends on whether Han got a diamond. It is either $\frac{12}{51}$ or $\frac{13}{51}$.

c. $\frac{1}{17}$ (or equivalent)

d. They are dependent events because Han's choice affects the probability of Jada's choice.

7 from Unit 8, Lesson 6

Student Task Statement

An agriculturist takes 50 samples of soil and measures the levels of two nutrients, nitrogen and phosphorus. In 46% of the samples the nitrogen levels are low and in 28% of the samples the phosphorus levels are low. In 10% of the samples both the nitrogen and the phosphorus levels are low. What percentage of the samples have nitrogen levels or phosphorus levels that are low?

Solution

64%

8 from Unit 8, Lesson 1

Student Task Statement

Select **all** of the situations that have a 50% chance of occurring.

A. Rolling a standard number cube and getting a 3.

B. Flipping two fair coins and getting heads on exactly one of the flips.

C. Picking a letter at random from the word SEED and getting an E.

D. Picking a letter at random from the word ORCHID and getting a vowel.

E. Getting the answer correct when guessing randomly on a true or false question.

Solution

B, C, E

