Unit 5, Lesson 3

Solving Quadratic Equations by Reasoning

Let's find solutions to quadratic equations.

3.1 How Many Solutions?

How many solutions does each equation have? What are the solution(s)? Be prepared to explain how you know.

1. $x^2 = 9$
2. $x^2 = 0$
3. $x^2 - 1 = 3$
4. $2x^2 = 50$
5. $(x + 1)(x + 1) = 0$
6. $x(x - 6) = 0$
7. $(x - 1)(x - 1) = 4$

3.2 Finding Pairs of Solutions

Each of these equations has two solutions. What are they? Explain or show your reasoning.

1. $n^2 + 4 = 404$

2. $432 = 3n^2$

3. $144 = (n + 1)^2$

4. $(n - 5)^2 - 30 = 70$

Are you ready for more?

1. How many solutions does the equation $(x - 3)(x + 1)(x + 5) = 0$ have? What are the solutions?

2. How many solutions does the equation $(x - 2)(x - 7)(x - 2) = 0$ have? What are the solutions?

3. Write a new equation that has 10 solutions.

Lesson 3 Summary

Some quadratic equations can be solved by performing the same operation on each side of the equal sign and reasoning about which values for the variable would make the equation true.

Suppose we wanted to solve $3(x+1)^2 - 75 = 0$. We can proceed like this:

- Add 75 to each side: $3(x+1)^2 = 75$
- Divide each side by 3: $(x+1)^2 = 25$
- What number can be squared to get 25? $(\square)^2 = 25$
- There are two numbers that work, 5 and -5: $5^2 = 25$ and $(\text{-}5)^2 = 25$
- If $x + 1 = 5$, then $x = 4$.
- If $x + 1 = \text{-}5$, then $x = \text{-}6$.

This means that both $x = 4$ and $x = \text{-}6$ make the equation true and are solutions to the equation.

Many quadratic equations have 2 solutions, but some have only 1 or no solution.