



# Completing the Square and Complex Solutions

Let's find complex solutions to quadratic equations by completing the square.

## 17.1 Creating Quadratic Equations

Match each equation in standard form to its factored form and its complex solutions.

- |                   |                                      |                         |
|-------------------|--------------------------------------|-------------------------|
| 1. $x^2 - 25 = 0$ | • $(x - 5i)(x + 5i) = 0$             | • $\sqrt{5}, -\sqrt{5}$ |
| 2. $x^2 - 5 = 0$  | • $(x - 5)(x + 5) = 0$               | • $5, -5$               |
| 3. $x^2 + 25 = 0$ | • $(x - \sqrt{5})(x + \sqrt{5}) = 0$ | • $5i, -5i$             |

## 17.2 Sometimes the Solutions Aren't Real Numbers

What are the complex solutions to these equations? Check your solutions by substituting them into the original equation.

1.  $(x - 5)^2 = 0$
2.  $(x - 5)^2 = 1$
3.  $(x - 5)^2 = -1$



## 17.3

## Finding Complex Solutions

Solve these equations by completing the square to find all complex solutions.

1.  $x^2 - 8x + 13 = 0$

2.  $x^2 - 8x + 19 = 0$

**Are you ready for more?**

For which values of  $a$  does the equation  $x^2 - 8x + a = 0$  have two real solutions? One real solution? No real solutions? Explain your reasoning.

## 17.4

## Can You See the Solutions on a Graph?

1. How many real solutions does each equation have? How many non-real solutions?

a.  $x^2 - 8x + 13 = 0$

b.  $x^2 - 8x + 16 = 0$

c.  $x^2 - 8x + 19 = 0$

2. How do the graphs of these functions help us answer the previous question?

a.  $f(x) = x^2 - 8x + 13$

b.  $g(x) = x^2 - 8x + 16$

c.  $h(x) = x^2 - 8x + 19$



## Lesson 17 Summary

Sometimes quadratic equations have real solutions, and sometimes they do not. Here is a quadratic equation with  $x^2$  equal to a negative number (assume  $k$  is positive):

$$x^2 = -k$$

This equation has imaginary solutions  $i\sqrt{k}$  and  $-i\sqrt{k}$ . By similar reasoning, an equation of the form:

$$(x - h)^2 = -k$$

has non-real solutions if  $k$  is positive. In this case, the solutions are  $h + i\sqrt{k}$  and  $h - i\sqrt{k}$ .

It isn't always clear just by looking at a quadratic equation whether the solutions will be real or not. For example, look at this quadratic equation:

$$x^2 - 12x + 41 = 0$$

We can always complete the square to find out what the solutions will be:

$$\begin{aligned}x^2 - 12x + 36 + 5 &= 0 \\(x - 6)^2 + 5 &= 0 \\(x - 6)^2 &= -5 \\x - 6 &= \pm i\sqrt{5} \\x &= 6 \pm i\sqrt{5}\end{aligned}$$

This equation has non-real, complex solutions  $6 + i\sqrt{5}$  and  $6 - i\sqrt{5}$ .