Proving the Triangle Congruence Theorems
Sentence Frames for Proofs

Transformations:

- Translate _______ from _______ to _______.
- Rotate _______ using _______ as the center by angle _______.
- Rotate _______ using _______ as the center so that _______ coincides with _______.
- Reflect _______ across _______.
- Reflect _______ across the perpendicular bisector of _______.
- Segments _______ and _______ are the same length so they are congruent. Therefore, there is a rigid motion that takes _______ to _______. Apply that rigid motion to _______.

Justifications:

- We know the image of _______ is congruent to _______ because rigid motions preserve measure.
- Points _______ and _______ coincide after translating because we defined our translation that way!
- Since points _______ and _______ are the same distance along the same ray from _______ they have to be in the same place.
- Rays _______ and _______ coincide after rotating because we defined our rotation that way!
- The image of _______ must be on ray _______ since both _______ and _______ are on the same side of _______ and make the same angle with it at _______.
- Points _______ and _______ coincide because they are both at the intersection of the same lines, and 2 distinct lines can only intersect in 1 place.
- _______ is the perpendicular bisector of the segment connecting _______ and _______, because the perpendicular bisector is determined by 2 points that are both equidistant from the endpoints of a segment.

Conclusion statement:

- We have shown that a rigid motion takes _______ to _______, _______ to _______, and _______ to _______, therefore triangle _______ is congruent to triangle _______.

What Do We Know For Sure About Isosceles Triangles?

Kiran

Kiran: I'm stumped on this proof.
Mai: What are you trying to prove?

Kiran: I'm trying to prove that in an isosceles triangle, the two base angles are congruent. So in this case, that angle A is congruent to angle B.
Mai: Let's think of what geometry ideas we already know are true.

Kiran: We know if two pairs of corresponding sides, and the corresponding angles between the sides, are congruent, then the triangles must be congruent.
Mai: Yes, and we also know that we can use reflections, rotations, and translations to prove congruence and symmetry. . . The isosceles triangle you've drawn makes me think of symmetry. If you draw a line down the middle of it, I wonder if that could help us prove that the angles are the same?
[Mai draws the line of symmetry of the triangle and labels the intersection of AB and the line of symmetry Q].

Kiran: Wait, when you draw the line, it breaks the triangle into two smaller triangles. I wonder if I could prove those triangles are congruent using Side-Angle-Side Congruence?
Mai: It's an isosceles triangle, so we know that one pair of corresponding sides is congruent. [Mai marks the congruent sides.]

Kiran: And this segment in the middle here is part of both triangles, so it has to be the same length for both. Look.

[**Kiran draws** the two halves of the isosceles triangle and marks the shared sides as congruent.]
Mai: So we have two pairs of corresponding sides that are congruent. How do we know the angles between them are congruent?

Kiran: I'm not sure. Maybe it has to do with how we drew that line of symmetry?

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