



# Rewriting Quadratic Expressions in Factored Form (Part 3)

Let's look closely at some special kinds of factors.

## 8.1

## Math Talk: Products of Large-ish Numbers

Evaluate mentally.

- $9 \cdot 11$
- $19 \cdot 21$
- $99 \cdot 101$
- $119 \cdot 121$

## 8.2

## Can Products Be Written as Differences?

1. Clare claims that  $(10 + 3)(10 - 3)$  is equivalent to  $10^2 - 3^2$  and  $(20 + 1)(20 - 1)$  is equivalent to  $20^2 - 1^2$ . Do you agree? Show your reasoning.
  
2. a. Use your observations from the first question to evaluate  $(100 + 5)(100 - 5)$ . Show your reasoning.
  
- b. Check your answer by computing  $105 \cdot 95$ .

3. Is  $(x + 4)(x - 4)$  equivalent to  $x^2 - 4^2$ ? Support your answer:

With a diagram:

Without a diagram:

	$x$	4
$x$		
-4		

4. Is  $(x + 4)^2$  equivalent to  $x^2 + 4^2$ ? Support your answer, either with or without a diagram.



### Are you ready for more?

1. Explain how your work in the previous questions can help you mentally evaluate  $22 \cdot 18$  and  $45 \cdot 35$ .
2. Here is a shortcut that can be used to mentally square any two-digit number. Let's take  $83^2$ , for example.
  - 83 is  $80 + 3$ .
  - Compute  $80^2$  and  $3^2$ , which give 6,400 and 9. Add these values to get 6,409.
  - Compute  $80 \cdot 3$ , which is 240. Double it to get 480.
  - Add 6,409 and 480 to get 6,889.

Try using this method to find the squares of some other two-digit numbers. (With some practice, it is possible to get really fast at this!) Then, explain why this method works.

## 8.3

## What If There Is No Linear Term?

Each row has a pair of equivalent expressions.

Complete the table.

If you get stuck, consider drawing a diagram.

(Heads up: One of them is impossible.)

factored form	standard form
$(x - 10)(x + 10)$	
$(2x + 1)(2x - 1)$	
$(4 - x)(4 + x)$	
	$x^2 - 81$
	$49 - y^2$
	$9z^2 - 16$
	$25t^2 - 81$
$(c + \frac{2}{5})(c - \frac{2}{5})$	
	$\frac{49}{16} - d^2$
$(x + \sqrt{5})(x - \sqrt{5})$	
	$x^2 - 6$
	$x^2 + 100$

## Lesson 8 Summary

Sometimes expressions in standard form don't have a linear term. Can they still be written in factored form?

Let's take  $x^2 - 9$  as an example. To help us write it in factored form, we can think of it as having a linear term with a coefficient of 0:  $x^2 + 0x - 9$ .

We know that we need to find two numbers that multiply to make -9 and add up to 0. The numbers 3 and -3 meet both requirements, so the factored form is  $(x + 3)(x - 3)$ .

To check that this expression is indeed equivalent to  $x^2 - 9$ , we can expand the factored expression by applying the distributive property:  $(x + 3)(x - 3) = x^2 - 3x + 3x + (-9)$ . Adding  $-3x$  and  $3x$  gives 0, so the expanded expression is  $x^2 - 9$ .

In general, a quadratic expression that is a difference of two squares and has the form  $a^2 - b^2$  can be rewritten as  $(a + b)(a - b)$ .

Here is a more complicated example:  $49 - 16y^2$ . This expression can be written as  $7^2 - (4y)^2$ , so an equivalent expression in factored form is  $(7 + 4y)(7 - 4y)$ .

What about  $x^2 + 9$ ? Can it be written in factored form?

Let's think about this expression as  $x^2 + 0x + 9$ . Can we find two numbers that multiply to make 9 and add up to 0? Here are factors of 9 and their sums:

- 9 and 1, sum: 10
- -9 and -1, sum: -10
- 3 and 3, sum: 6
- -3 and -3, sum: -6

For two numbers to add up to 0, they need to be opposites (a negative and a positive), but a pair of opposites cannot multiply to make positive 9, because multiplying a negative number and a positive number always gives a negative product.

Because there are no numbers that multiply to make 9 and also add up to 0, it is not possible to write  $x^2 + 9$  in factored form using the kinds of numbers that we know about.