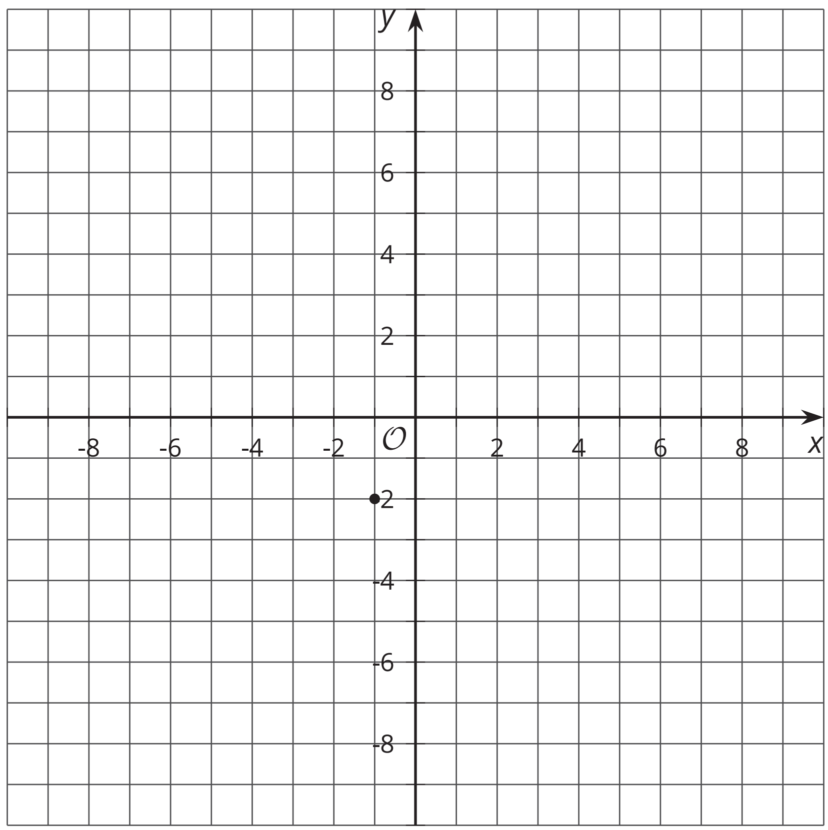
Connecting Distance and Circles

Let's look at points a given distance away from a particular point.

1.1 A Distance Away

Plot as many points as you can that are a distance of 5 away from the point $(-1, -2)$.



1.2 Plot a Distance Away

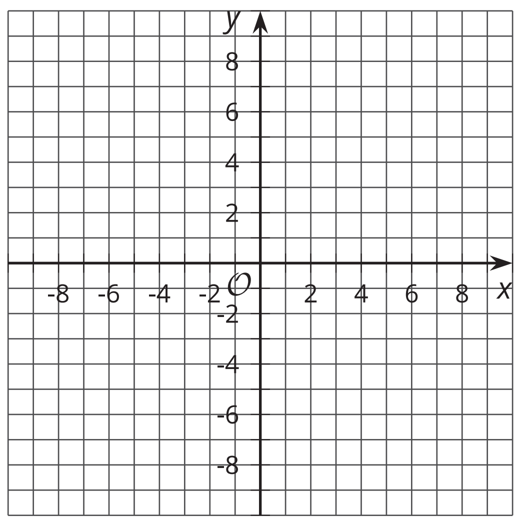
Here is a list of points.

(0, 8.5)   (0, -6.5)   (-10, 0)   (2.5, 6)   (6, 8)   (-4, -7.5)   (-4, 7.5)   (-6, -8)   (-2.5, -6)

(7.5, 4)   (8, 6)   (-6, 2.5)   (-6, -2.5)   (-8, 6)   (7.5, -4)

1. Sort the points according to their distance from the origin.

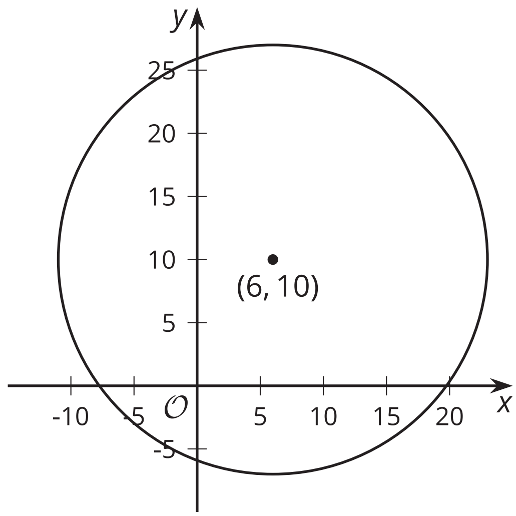
2. Your teacher will assign you to work with the points that are 6.5 units, 8.5 units, or 10 units from the origin.
   a. Plot the points on the coordinate plane.



   b. Find at least 2 more points the same distance from the origin that do not lie on either the x- or y-axis, and plot them on the plane.
   c. Use a compass to draw a circle centered at the origin with a radius that is the same as the distance you were assigned.

1.3 Circling the Problem

The image shows a circle with its center at $(6, 10)$ and radius of 17 units.



1. The point $(14, 25)$ looks like it might be on the circle. Verify if it really is on the circle. Explain or show your reasoning.

2. The point $(22, 3)$ looks like it might be on the circle. Verify if it really is on the circle. Explain or show your reasoning.

3. In general, how can you check if a particular point (x, y) is on the circle?

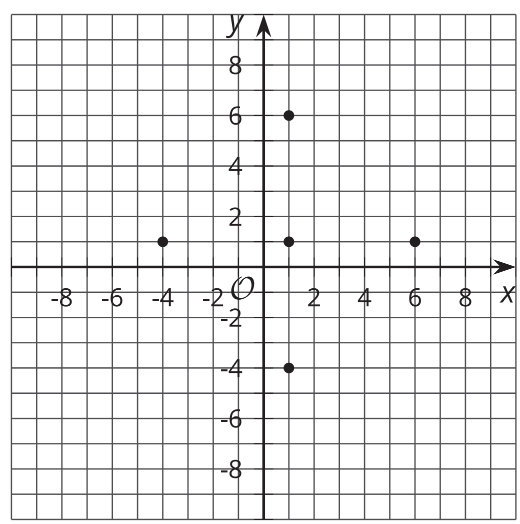
Are you ready for more?

Circle P has a center at point P at $(-4, 4)$ and a radius of 5. Circle Q has a center at point Q at $(2, 4)$ and a radius of 5.

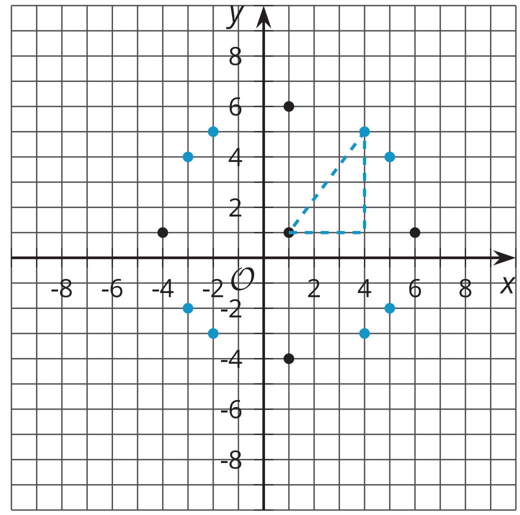
1. Graph the circles. Include points P and Q.
2. Draw the perpendicular bisector of PQ. How do you know it is the perpendicular bisector?
3. Find the points of intersection of the circles.

Lesson 1 Summary

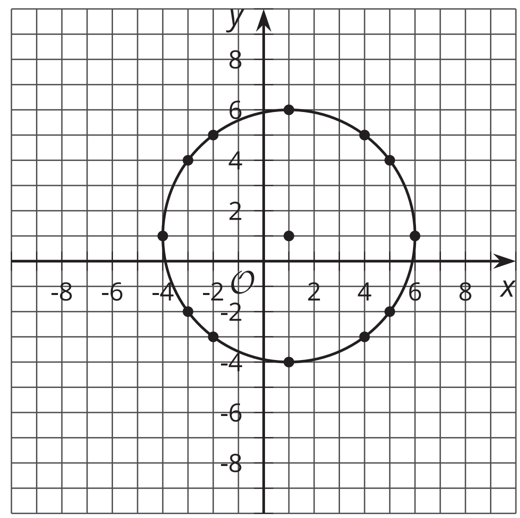
Let's find some points that are a distance of 5 away from the point $(1, 1)$ and plot them on a grid. We can use gridlines to find 4 points:



We can use the Pythagorean Theorem to help us find some more points. Recall that for a right triangle with a hypotenuse of 5, we can make a right triangle with legs 3 and 4. This means that if the horizontal and vertical distances between the points are 3 and 4, then the distance between them will be 5. We can find 8 more points this way!



As we fill in more points, it looks like the points are forming a circle around the point $(1, 1)$, and in fact, they are. The circle has a radius of 5, which makes sense since we know that a circle is the set of all points a given distance away from a central point. In this case, that given distance is 5.



How can we check whether a point lies on this circle if we are not sure? Let's check the point $(3, \text{-}3.5)$. We can use the Pythagorean Theorem to test the distance between the center of the circle and the point: $\sqrt{(3-1)^2 + (\text{-}3.5-1)^2} = 4.38$, which is not equal to 5. That means the point is close to the circle, but does not lie on the circle.

We can say that for a circle of radius r and center C, if a point P is a distance of r away from C, it will lie on the circle. If point P is not a distance of r away from C, then it will not lie on the circle. Another way to say this is that for a circle of radius r and center C, P will lie on the circle if and only if it is a distance of r away from C.

