



# Let's Make a Box

Let's investigate volumes of different boxes.

## 1.1

### Which Three Go Together: Boxes

Which three go together? Why do they go together?

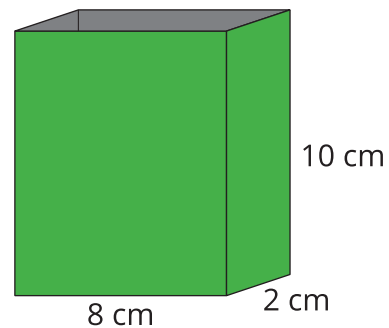
**A**

length: 4cm

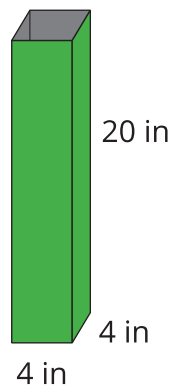
width: 8cm

height: 10cm

**B**

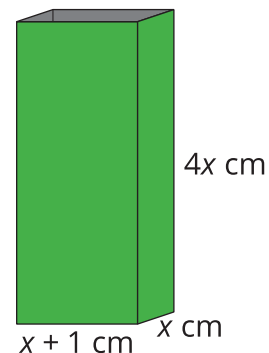


**C**



**D**

$$V = 320 \text{ cm}^3$$



1.2

Building Boxes

Your teacher will give you some supplies to construct an open-top box.

- 1. Cut out a square from each corner of a sheet of paper, and then fold up the sides.
- 2. Calculate the volume of your box, and complete the table with your information.

| side length of square cutout (in) | length (in) | width (in) | height (in) | volume of box (in <sup>3</sup> ) |
|-----------------------------------|-------------|------------|-------------|----------------------------------|
| 1                                 |             |            |             |                                  |
|                                   |             |            |             |                                  |
|                                   |             |            |             |                                  |

1.3

Building the Biggest Box

- 1. The volume  $V(x)$  in cubic inches of the open-top box is a function of the side length  $x$  in inches of the square cutouts. Make a plan to figure out how to construct the box with the largest volume.



Pause here so your teacher can review your plan.

- 2. Write an expression for  $V(x)$ .
- 3. Use graphing technology to create a graph representing  $V(x)$ . Approximate the value of  $x$  that would allow you to construct an open-top box with the largest volume possible from one piece of paper.



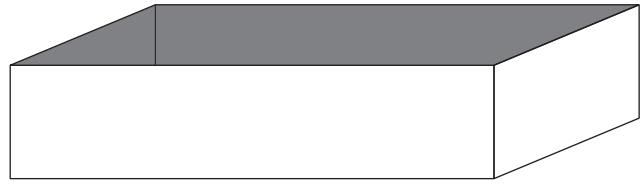
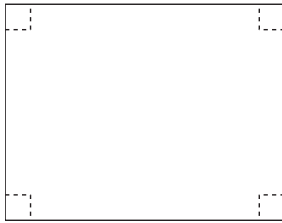
### Are you ready for more?

The surface area  $A(x)$ , in square inches, of the open-top box is also a function of the side length  $x$ , in inches, of the square cutouts.

1. Find one expression for  $A(x)$  by adding the area of the five faces of our open-top box.
2. Find another expression for  $A(x)$  by subtracting the area of the cutouts from the area of the paper.
3. Show algebraically that these two expressions are equivalent.

## Lesson 1 Summary

A box can be created by removing squares from each corner of a rectangle of paper.



Let  $V(x)$  be the volume of the box in cubic inches, where  $x$  is the side length, in inches, of each square removed from the four corners.

To define  $V$  using an expression, we can use the fact that the volume of a cube is  $(length)(width)(height)$ . If the piece of paper we start with is 3 inches by 8 inches, then:

$$V(x) = (3 - 2x)(8 - 2x)(x)$$

What are some reasonable values for  $x$ ? Cutting out squares with side lengths less than 0 inches doesn't make sense, and similarly, we can't cut out squares larger than 1.5 inches, since the short side of the paper is only 3 inches (since  $3 - 1.5 \cdot 2 = 0$ ). You may remember that the name for the set of all the input values that make sense to use with a function is the domain. Here, a reasonable domain is somewhere larger than 0 inches but less than 1.5 inches, depending on how well we can cut and fold!

By graphing this function, it is possible to find the maximum value within a specific domain. Here is a graph of  $y = V(x)$  for  $x$  values between 0 and 1.5. It looks like the largest volume we can get for a box made this way from a 3-inch by 8-inch piece of paper is about  $7.4 \text{ in}^3$ .

