



# What about Other Bases?

Let's explore exponent patterns with bases other than 10.

## 5.1 Math Talk: Comparing Expressions with Exponents

Decide mentally whether each statement is true.

- $3^5 < 4^6$
- $(-3)^2 < 3^2$
- $(-3)^3 = 3^3$
- $(-5)^2 > -5^2$



## 5.2

## What Happens with Zero and Negative Exponents?

Complete the table.

|       |       |  |  |  |  |               |  |  |  |
|-------|-------|--|--|--|--|---------------|--|--|--|
| value | 16    |  |  |  |  | $\frac{1}{2}$ |  |  |  |
|       | $2^4$ |  |  |  |  |               |  |  |  |

Diagram showing arrows above the table indicating multiplication by 2 from right to left, and arrows below the table indicating division by 2 from left to right.

- As you move toward the left, each number is being multiplied by 2. What is the multiplier as you move toward the right?
- Use the patterns you found in the table to write  $2^{-6}$  as a fraction.
- Write  $\frac{1}{32}$  as a power of 2 with a single exponent.
- What is the value of  $2^0$ ?
- From the work you have done with negative exponents, how would you write  $5^{-3}$  as a fraction?
- How would you write  $3^{-4}$  as a fraction?



### Are you ready for more?

1. Find an expression equivalent to  $\left(\frac{2}{3}\right)^{-3}$  but with positive exponents.
2. Find an expression equivalent to  $\left(\frac{4}{5}\right)^{-8}$  but with positive exponents.
3. What patterns do you notice when you start with a fraction raised to a negative exponent and rewrite it using a single positive exponent? Show or explain your reasoning.



## 5.3

## Exponent Rules with Bases Other than 10

Lin, Noah, Diego, and Elena each chose an expression to start with and then came up with a new list of expressions — some of which are equivalent to the original and some of which are not.

Choose 2 of the 4 lists to analyze. For each list of expressions you choose to analyze, decide which expressions are *not* equivalent to the original. Be prepared to explain your reasoning.

1. Lin's original expression is  $5^{-9}$  and her list is:

$$(5^3)^{-3}$$

$$-5^9$$

$$\frac{5^{-6}}{5^3}$$

$$(5^3)^{-2}$$

$$\frac{5^{-4}}{5^{-5}}$$

$$5^{-4} \cdot 5^{-5}$$

2. Noah's original expression is  $3^{10}$  and his list is:

$$3^5 \cdot 3^2$$

$$(3^5)^2$$

$$(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)$$

$$\left(\frac{1}{3}\right)^{-10}$$

$$3^7 \cdot 3^3$$

$$\frac{3^{20}}{3^{10}}$$

$$\frac{3^{20}}{3^2}$$

3. Diego's original expression is  $x^4$  and his list is:

$$\frac{x^8}{x^4}$$

$$x \cdot x \cdot x \cdot x$$

$$\frac{x^{-4}}{x^{-8}}$$

$$\frac{x^{-4}}{x^8}$$

$$(x^2)^2$$

$$4 \cdot x$$

$$x \cdot x^3$$

4. Elena's original expression is  $8^0$  and her list is:

$$1$$

$$0$$

$$8^3 \cdot 8^{-3}$$

$$\frac{8^2}{8^2}$$

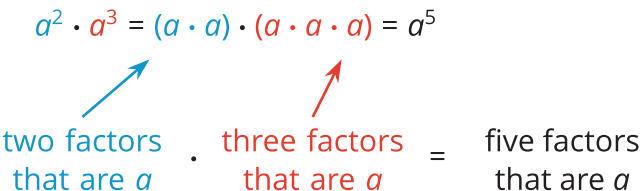
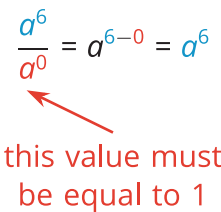
$$10^0$$

$$11^0$$



## Lesson 5 Summary

We can keep track of repeated factors using exponent rules. These rules also help us make sense of negative exponents and why a number to the power of 0 is defined as 1. These rules can be written symbolically where the base  $a$  can be any positive number:

| Rule                        | Example showing how it works                                                                                                                                                                                                                                                                                 |
|-----------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $a^n \cdot a^m = a^{n+m}$   | $a^2 \cdot a^3 = (a \cdot a) \cdot (a \cdot a \cdot a) = a^5$  <p>two factors that are <math>a</math> <math>\cdot</math> three factors that are <math>a</math> = five factors that are <math>a</math></p>                  |
| $(a^n)^m = a^{n \cdot m}$   | $(a^2)^3 = \underbrace{(a \cdot a)}_{\text{two factors that are } a} \cdot \underbrace{(a \cdot a)}_{\text{two factors that are } a} \cdot \underbrace{(a \cdot a)}_{\text{two factors that are } a} = a^6$ <p>three groups of two factors that are <math>a</math> = six factors that are <math>a</math></p> |
| $\frac{a^n}{a^m} = a^{n-m}$ | $\frac{a^5}{a^2} = \frac{a \cdot a \cdot a \cdot a \cdot a}{a \cdot a} = \frac{a \cdot a}{a \cdot a} \cdot a \cdot a \cdot a = 1 \cdot a^3 = a^3$ <p>five factors that are <math>a</math> <math>\div</math> two factors that are <math>a</math> = three factors that are <math>a</math></p>                  |
| $a^0 = 1$                   | $\frac{a^6}{a^0} = a^{6-0} = a^6$  <p>this value must be equal to 1</p>                                                                                                                                                  |
| $a^{-n} = \frac{1}{a^n}$    | $a^{-3} = \frac{1}{a} \cdot \frac{1}{a} \cdot \frac{1}{a} = \frac{1}{a^3}$ <p>three factors that are one over <math>a</math></p>                                                                                                                                                                             |