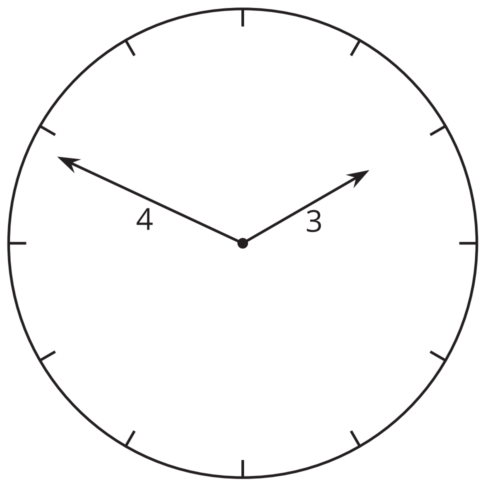
The Converse

Let's figure out if a triangle is a right triangle.

8.1 The Hands of a Clock

Consider the tips of the hands of an analog clock that has an hour hand that is 3 centimeters long and a minute hand that is 4 centimeters long.



Over the course of a day:

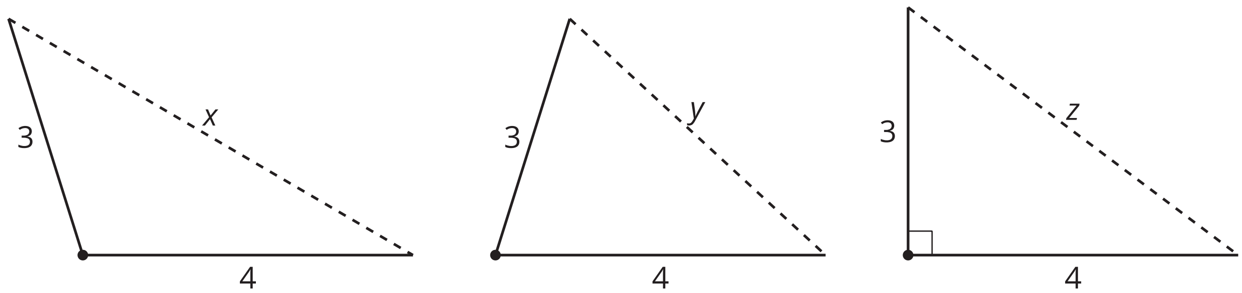
1. What is the farthest distance apart the two tips get?

2. What is the closest distance the two tips get?

3. Are the two tips ever exactly five centimeters apart? Explain your reasoning.

8.2 Proving the Converse

Here are three triangles with two side lengths measuring 3 and 4 units, and the third side of unknown length.



Order the following six values from smallest to largest. Put an equal sign between any you know to be equal. Be prepared to explain your reasoning.

$1 \quad 5 \quad 7 \quad x \quad y \quad z$

Are you ready for more?

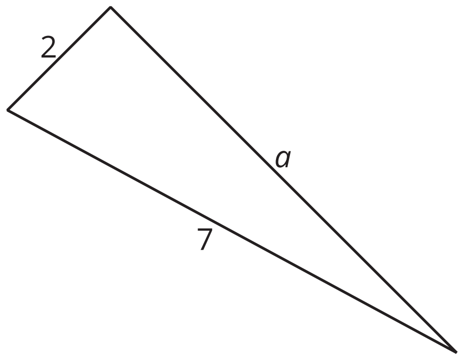
A related argument also lets us distinguish acute from obtuse triangles using only their side lengths.

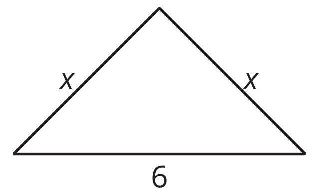
Decide if triangles with the following side lengths are acute, right, or obtuse. In right or obtuse triangles, identify which side length is opposite the right or obtuse angle.

1. $x = 15, y = 20, z = 8$
2. $x = 8, y = 15, z = 13$
3. $x = 17, y = 8, z = 15$

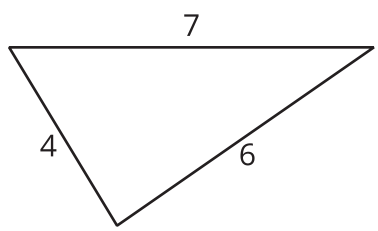
8.3 Calculating Legs of Right Triangles

1. Given the information provided for the right triangles shown here, find the unknown leg lengths to the nearest tenth.



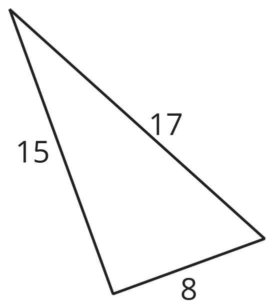


2. The triangle shown here is not a right triangle. What are two different ways you change *one* of the values so it would be a right triangle? Sketch these new right triangles, and clearly label the right angle.



Lesson 8 Summary

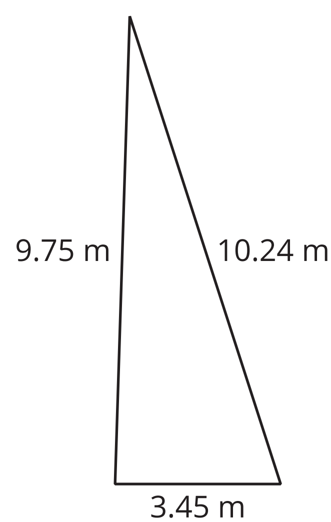
How can we tell whether a triangle is a right triangle or not? For example, in this triangle it isn't clear just by looking, and it may be that the sides aren't drawn to scale.



If we have a triangle with side lengths a, b, and c, with c being the longest of the three, then the converse of the Pythagorean Theorem tells us that any time we have $a^2 + b^2 = c^2$, we must have a right triangle. Since $8^2 + 15^2 = 64 + 225 = 289 = 17^2$, any triangle with side lengths 8, 15, and 17 must be a right triangle.

What about the jib sail on this boat? It is a triangle, but is it a right triangle?

The measurements of the jib sail are shown here. The sum of the squares of the two shorter sides is 106.965 square meters, and the square of the longest side is 104.8576 square meters. So by the converse of the Pythagorean Theorem, it is not a right triangle, but it is close to one.



Together, the Pythagorean Theorem and its converse provide a way to check if a triangle is a right triangle just using its side lengths. If $a^2 + b^2 = c^2$, it is a right triangle. If $a^2 + b^2 \neq c^2$, it is not a right triangle.