Lots of Lines (Part 1)

Narrator: Diego, Jada, and Noah were given the following task: "Prove that if a point *C* is the same distance from *A* as it is from *B*, then *C* must be on the perpendicular bisector of *AB*." At first they were really stuck.

Noah: How do you prove a point is on a line?

Narrator: Their teacher gave them the hint, "Another way to think about it is to draw a line that you know *C* is on, and prove that line has to be the perpendicular bisector." They each drew a line and thought about their pictures.

Diego: I drew a line through *C* that was perpendicular to *AB* and through the midpoint of *AB*. That line is the perpendicular bisector of *AB* and *C* is on it, so that proves *C* is on the perpendicular bisector."

Jada: I thought the line through *C* would probably go through the midpoint of *AB* so I drew that and labeled the midpoint *D*. Triangle *ACB* is isosceles, so angles *A* and *B* are congruent, and *AC* and *BC* are congruent. And *AD* and *DB* are congruent because *D* is a midpoint. That made two congruent triangles by the Side-Angle-Side Triangle Congruence Theorem. So I know angle *ADC* and angle *BDC* are congruent, but I still don't know if *DC* is the perpendicular bisector of *AB*.

Noah: In the Isosceles Triangle Theorem proof, Mai and Kiran drew an angle bisector in their isosceles triangle, so I'll try that. I'll draw the angle bisector of angle *ACB*. The point where the angle bisector hits *AB* will be *D*. So triangles *ACD* and *BCD* are congruent, which means *AD* and *BD* are congruent, so *D* is a midpoint and *CD* is the perpendicular bisector.

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