Unit 9, Lesson 12

Reasoning about Exponential Graphs (Part 1)

Let's study and compare equations and graphs of exponential functions

12.1 Spending Gift Money

Jada receives a gift of $180. In the first week, she spends a third of the gift money. She continues spending a third of what is left each week thereafter. Which equation best represents the amount of gift money g, in dollars, she has after t weeks? Be prepared to explain your reasoning.

A. $g = 180 - \frac{1}{3}t$

B. $g = 180 \cdot \left(\frac{1}{3}\right)^t$

C. $g = \frac{1}{3} \cdot 180^t$

D. $g = 180 \cdot \left(\frac{2}{3}\right)^t$

12.2 Equations and Their Graphs

1. Each of the functions f, g, h, and j represents the amount of money in a bank account, in dollars, as a function of time x, in years. They are each written in form $m(x) = a \cdot b^x$.

$$f(x) = 50 \cdot 2^x$$

$$g(x) = 50 \cdot 3^x$$

$$h(x) = 50 \cdot \left(\frac{3}{2}\right)^x$$

$$j(x) = 50 \cdot (0.5)^x$$

   a. Use graphing technology to graph each function on the same coordinate plane.

   b. Explain how changing the value of b changes the graph.

2. Here are equations defining functions p, q, and r. They are also written in the form $m(x) = a \cdot b^x$.

$$p(x) = 10 \cdot 4^x$$

$$q(x) = 40 \cdot 4^x$$

$$r(x) = 100 \cdot 4^x$$

   a. Use graphing technology to graph each function and to update your choice of accounts, if needed.

   b. Explain how changing the value of a changes the graph.

Are you ready for more?

As before, consider bank accounts whose balances are given by the following functions:

$$f(x) = 10 \cdot 3^x \qquad g(x) = 3^{x+2} \qquad h(x) = \tfrac{1}{2} \cdot 3^{x+3}$$

Which function would you choose? Does your choice depend on x?

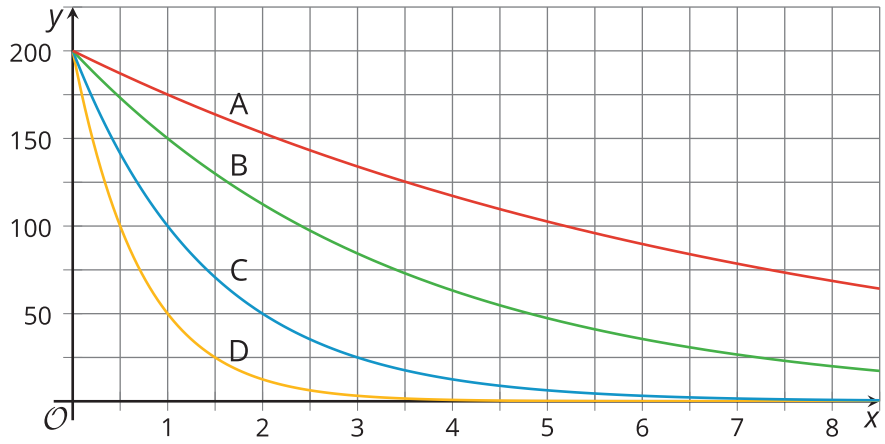
12.3 Graphs Representing Exponential Decay

$$m(x) = 200 \cdot \left(\frac{1}{4}\right)^x$$

$$n(x) = 200 \cdot \left(\frac{1}{2}\right)^x$$

$$p(x) = 200 \cdot \left(\frac{3}{4}\right)^x$$

$$q(x) = 200 \cdot \left(\frac{7}{8}\right)^x$$



1. Match each equation with a graph. Be prepared to explain your reasoning.
2. Functions f and g are defined by these two equations: $f(x) = 1{,}000 \cdot \left(\frac{1}{10}\right)^x$ and $g(x) = 1{,}000 \cdot \left(\frac{9}{10}\right)^x$.

   a. Which function is decaying more quickly? Explain your reasoning.

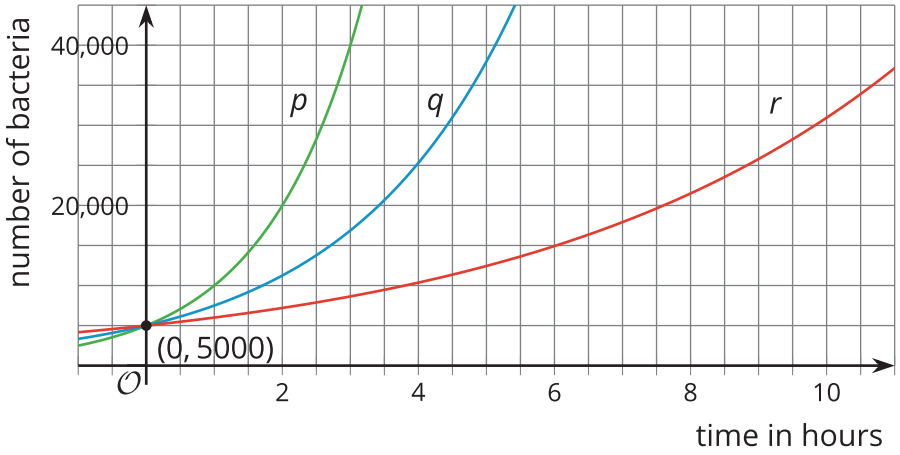
   b. Use graphing technology to verify your response.

Lesson 12 Summary

An exponential function can give us information about a graph that represents it.

For example, suppose that function q represents a bacteria population t hours after it is first measured, and $q(t) = 5{,}000 \cdot (1.5)^t$. The number 5,000 is the bacteria population measured, when t is 0. The number 1.5 indicates that the bacteria population increases by a factor of 1.5 each hour.

A graph can help us see how the starting population (5,000) and growth factor (1.5) influence the population. Suppose functions p and r represent two other bacteria populations and are given by $p(t) = 5{,}000 \cdot 2^t$ and $r(t) = 5{,}000 \cdot (1.2)^t$. Here are the graphs of p, q, and r.



All three graphs start at $5{,}000$, but the graph of r grows more slowly than does the graph of q, while the graph of p grows more quickly. This makes sense because a population that doubles every hour is growing more quickly than one that increases by a factor of 1.5 each hour, and both grow more quickly than a population that increases by a factor of 1.2 each hour.