## Lesson 15: Weighted Averages

* Let’s split segments using averages and ratios.

### 15.1: Part Way: Points

For the questions in this activity, use the coordinate grid if it is helpful to you.



1. What is the midpoint of the segment connecting $\left(1,2\right)$ and $\left(5,2\right)$?
2. What is the midpoint of the segment connecting $\left(5,2\right)$ and $\left(5,10\right)$?
3. What is the midpoint of the segment connecting $\left(1,2\right)$ and $\left(5,10\right)$?

### 15.2: Part Way: Segment

Point $A$ has coordinates $\left(2,4\right)$. Point $B$ has coordinates $\left(8,1\right)$.



1. Find the point that partitions segment $AB$ in a $2:1$ ratio.
2. Calculate $C=\frac{1}{3}A+\frac{2}{3}B$.
3. What do you notice about your answers to the first 2 questions?
4. For 2 new points $K$ and $L$, write an expression for the point that partitions segment $KL$ in a $3:1$ ratio.

#### Are you ready for more?

Consider the general quadrilateral $QRST$ with $Q=\left(0,0\right),R=\left(a,b\right),S=\left(c,d\right),$ and $T=\left(e,f\right)$.

1. Find the midpoints of each side of this quadrilateral.
2. Show that if these midpoints are connected consecutively, the new quadrilateral formed is a parallelogram.

### 15.3: Part Way: Quadrilateral

Here is quadrilateral $ABCD$.



1. Find the point that partitions segment $AB$ in a $1:4$ ratio. Label it $B^{′}$.
2. Find the point that partitions segment $AD$ in a $1:4$ ratio. Label it $D^{′}$.
3. Find the point that partitions segment $AC$ in a $1:4$ ratio. Label it $C^{′}$.
4. Is $AB^{′}C^{′}D^{′}$ a dilation of $ABCD$? Justify your answer.

### Lesson 15 Summary

To find the midpoint of a line segment, we can average the coordinates of the endpoints. For example, to find the midpoint of the segment from $A=\left(0,4\right)$ to $B=\left(6,7\right)$, average the coordinates of $A$ and $B$: $\left(\frac{0+6}{2},\frac{4+7}{2}\right)=\left(3,5.5\right)$. Another way to write what we just did is $\frac{1}{2}\left(A+B\right)$ or $\frac{1}{2}A+\frac{1}{2}B$.

Now, let’s find the point that is $\frac{2}{3}$ of the way from $A$ to $B$. In other words, we’ll find point $C$ so that segments $AC$ and $CB$ are in a $2:1$ ratio.

In the horizontal direction, segment $AB$ stretches from $x=0$ to $x=6$. The distance from 0 to 6 is 6 units, so we calculate $\frac{2}{3}$ of 6 to get 4. Point $C$ will be 4 horizontal units away from $A$, which means an $x$-coordinate of 4.



In the vertical direction, segment $AB$ stretches from $y=4$ to $y=7$. The distance from 4 to 7 is 3 units, so we can calculate $\frac{2}{3}$ of 3 to get 2. Point $C$ must be 2 vertical units away from $A$, which means a $y$-coordinate of 6.

It is possible to do this all at once by saying $C=\frac{1}{3}A+\frac{2}{3}B$. This is called a weighted average. Instead of finding the point in the middle, we want to find a point closer to $B$ than to $A$. So we give point $B$ more weight—it has a coefficient of $\frac{2}{3}$ rather than $\frac{1}{2}$ as in the midpoint calculation. To calculate $C=\frac{1}{3}A+\frac{2}{3}B$, substitute and evaluate.

$\frac{1}{3}A+\frac{2}{3}B$

$\frac{1}{3}\left(0,4\right)+\frac{2}{3}\left(6,7\right)$

$\left(0,\frac{4}{3}\right)+\left(4,\frac{14}{3}\right)$

$\left(4,6\right)$

Either way, we found that the coordinates of $C$ are $\left(4,6\right)$.



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