Unit 1, Lesson 4

Parallelograms

Goals

- Compare and contrast (orally) different strategies for determining the area of a parallelogram.
- Describe (orally and in writing) observations about the opposite side and opposite angles of parallelograms.
- Explain (orally and in writing) how to find the area of a parallelogram by rearranging its parts or by enclosing it in a rectangle.

Learning Targets

- I can use reasoning strategies and what I know about the area of a rectangle to find the area of a parallelogram.
- I know how to describe the characteristics of a parallelogram using mathematical vocabulary.

Lesson Narrative

In this lesson, students recall the defining attributes of parallelograms and other properties that follow from that definition. Students then use reasoning strategies from earlier work to find areas of parallelograms.

One approach for finding the area of a parallelogram is to decompose the parallelogram and rearrange the pieces into a rectangle. Another is to enclose the parallelogram in a rectangle of the same height and then subtract the area of the extra regions—two right triangles that can be rearranged into a rectangle.

By working with various parallelograms, students begin to see that the shape of certain parallelograms may encourage the use of certain strategies. For instance, a parallelogram that is narrow and stretched out may be cumbersome to decompose and rearrange. Enclosing it in a rectangle and subtracting the areas of the two extra pieces might be preferable.

Through repeated reasoning, students begin to see regularity (MP8): parallelograms have related rectangles that can be used to find their area. Students also describe the process of finding the area of a parallelogram more generally, which prepares them to express that process as a formula.

A note about notation:

When recording students' solutions and reasoning in this lesson, consider using the "dot" notation instead of the "cross" notation to indicate multiplication. Explain that the $\cdot$ symbol and the $\times$ symbol both represent multiplication. Doing so familiarizes students with the use of the notation before they see it in student-facing materials.

Standards

Building On	**4.G.A.2, 5.G.B**
Addressing	**6.G.A.1**

Instructional Routines

- MLR7: Compare and Connect

Required Materials

Materials to Gather

- Geometry toolkits: Activity 1, Activity 2, Activity 3

Materials to Copy

- Area of a Parallelogram Cutouts (1 copy for every 1 students): Activity 2

Required Preparation

Activity 2:

For the digital version of the activity, acquire devices that can run the applet.

Student Facing Learning Goals

Let’s investigate the characteristics and area of parallelograms.

4.1 What Are Parallelograms?

Warm-up

5 min

Activity Narrative

In this activity, students examine examples and non-examples of parallelograms and identify their defining characteristics. Students recall that a parallelogram is a quadrilateral whose opposite sides are parallel. They observe other properties that follow from that definition—that opposite sides of a parallelogram have the same length and opposite angles have the same measure.

Standards

Building On **4.G.A.2, 5.G.B**

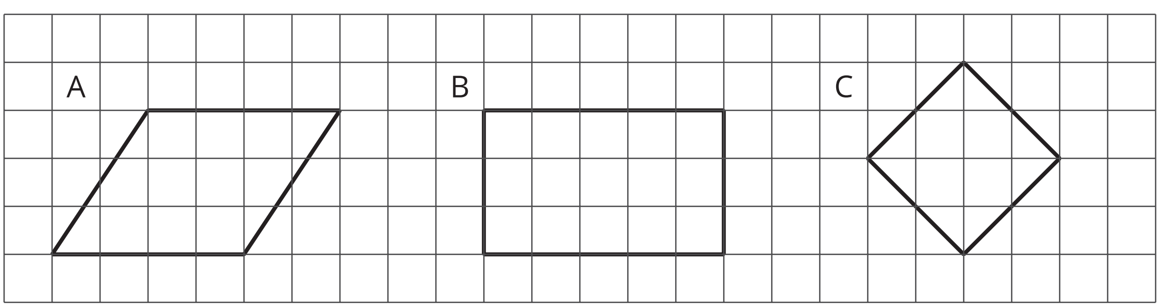
Launch

Display the images of Figures A–F for all to see. Tell students that Figures A, B, and C are parallelograms and Figures D, E, and F are not parallelograms.

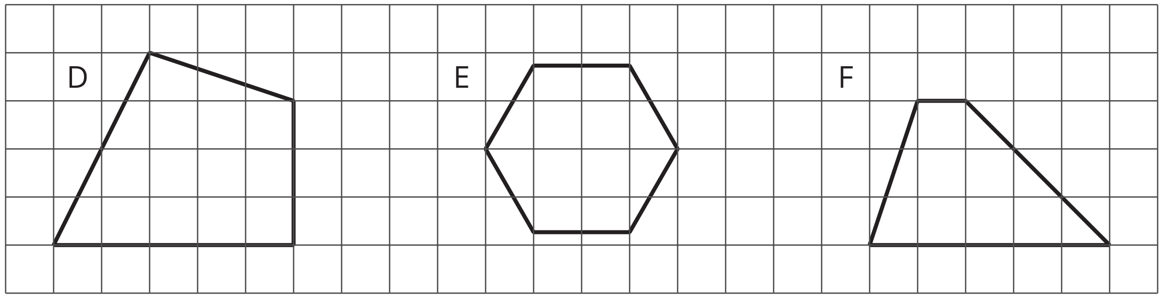
Arrange students into groups of 2 and provide access to geometry toolkits. Give students 1–2 minutes of quiet think time to complete the task. Afterward, give them a minute to discuss their answers and observations with their partner.

Student Task Statement

Figures A, B, and C are *parallelograms*.



Figures D, E, and F are not parallelograms.



What do you notice about:

1. The number of sides that a parallelogram has?
2. Opposite sides of a parallelogram?
3. Opposite angles of a parallelogram?

Student Response

1. Parallelograms have four sides.
2. Opposite sides of parallelograms are parallel and have the same length.
3. Opposite angles of parallelograms have the same measure.

Building on Student Thinking

Students may not realize that Figure C is a square or relate Figure C to the other parallelograms because of its orientation. Encourage students to use patty paper or another tool in the geometry toolkit to help them compare the characteristics of Figure C to those of Figures A and B.

Activity Synthesis

Ask a few students to share their responses to the questions. After each response, ask students to indicate whether they agree. If a student disagrees, discuss the disagreement. Record the agreed-upon responses for all to see and highlight these characteristics of *parallelograms*:

- A parallelogram is a quadrilateral (a polygon with four sides).
- Both pairs of its opposite sides are parallel.
- Its opposite sides have the same length.
- Its opposite angles have the same measure.

Students may wonder how to know if two non-horizontal or non-vertical sides of a figure are parallel. Explain that because parallel lines never intersect, the length of any perpendicular line segments between them are the same length. Consider demonstrating how to use an index card to check this in Figures A and C.

Tell students that for now we will just take these characteristics of parallelograms as facts. Later they will learn some ways to prove that these characteristics are always true.

4.2 Area of a Parallelogram

15 min

Activity Narrative

There is a digital version of this activity.

In this activity, students explore different methods of decomposing a parallelogram and rearranging the pieces to find its area. Here are two key approaches for finding the area of parallelograms:

- Decompose the parallelogram, rearrange the parts into a rectangle, and multiply the length of the base by the length of a side to find the area.
- Enclose the parallelogram in a rectangle and subtract the combined area of the extra regions.

Presenting the parallelograms on a grid makes it easier for students to see that the area does not change as they decompose and rearrange the pieces. This investigation lays a foundation for later reasoning about the area of triangles and other polygons.

Monitor for students who use the two key approaches and whose reasoning can highlight the usefulness of using a related rectangle to find the area of a parallelogram.

Some students may begin by counting squares but that strategy is not reinforced here. Encourage students to listen for and try more sophisticated, grade-appropriate methods shared during the class discussion.

In the digital version of the activity, students use an applet with some given rectangles and right triangles to visualize their reasoning (decomposition, rearrangement, and enclosure). The digital version may be helpful for visualizing the rearrangement of pieces to form rectangles.

Standards

Addressing **6.G.A.1**

Launch

Arrange students in groups of 2–4. Ask students to find the area of the parallelograms using recently learned strategies and tools. A blackline master with a larger version of the parallelograms is provided. Make copies of the blackline master available in case students wish to reason by cutting the parallelograms.

Give students 5 minutes of quiet think time and access to their toolkits. Ask them to share their strategies with their group afterward.

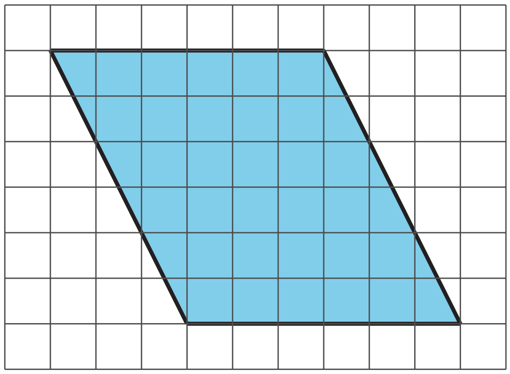
To encourage students to be mindful of their strategies and to create the foundation for the whole-class discussion, display and read aloud the following reflection questions before students begin working.

- “Why did you decompose the parallelogram the way you did?”
- “Why did you rearrange the pieces the way you did?”

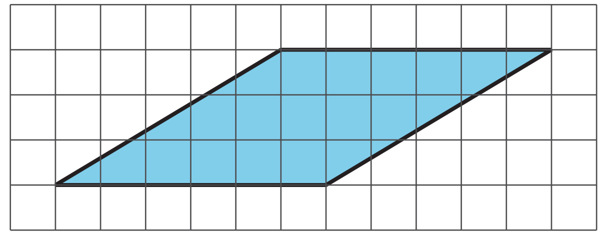
Student Task Statement

Find the area of each parallelogram. Show your reasoning.

1.

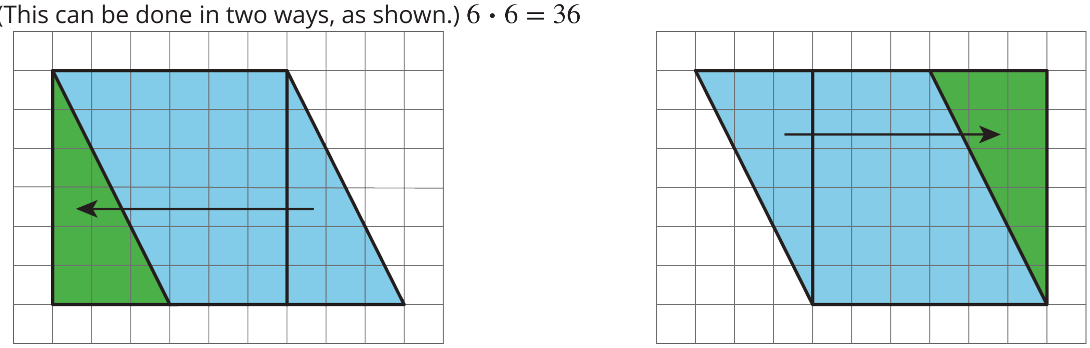


2.

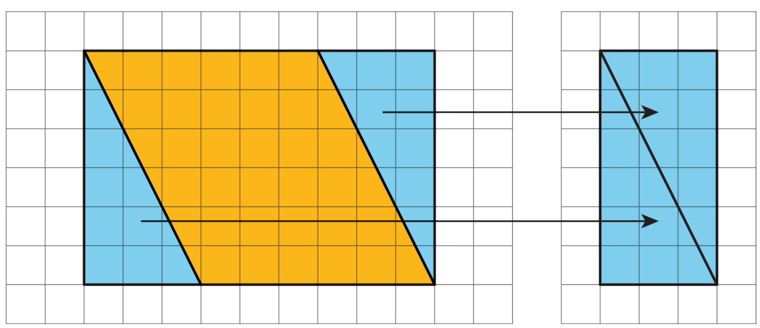


Student Response

1. 36 square units. Sample reasoning:
    - Decompose the parallelogram to create a right triangle and move it to form a rectangle that is 6 units by 6 units. (This can be done in two ways, as shown.) $6 \cdot 6 = 36$



    - Enclose the parallelogram within a rectangle and subtract the extra pieces. To subtract the area of the two right triangles, students may count the squares, or put them together to form a rectangle. $(6 \cdot 9) - (6 \cdot 3) = 36$.



2. 18 square units. Reasoning should involve decomposition and rearrangement, or other area reasoning strategies.

Activity Synthesis

Invite selected students to share how they found the area of the first parallelogram. Begin with students who decomposed the parallelogram in different ways. Follow with students who enclosed the parallelogram and rearranged the extra right triangles. (For students using the digital version, begin with those who reasoned with the smaller rectangles in the applet. Follow with students who reasoned with the largest rectangle.)

As students share, display and list the strategies for all to see. Restate them in terms of decomposing, rearranging, and enclosing, as needed. The list will serve as a reference for upcoming work. If one of the key strategies is not mentioned, illustrate it and add it to the list.

Use the reflection questions in the launch to help highlight the usefulness of rectangles in finding the area of parallelograms. Consider using the applet to illustrate this point, https://ggbm.at/kj5DcRvn.

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color coding and annotations to highlight connections between representations in a problem. For example, color code the diagrams that display each strategy used by students to find the area of the parallelograms. Label each diagram with the strategy shown (decomposing, rearranging, or enclosing).
Supports accessibility for: Visual-Spatial Processing

4.3 Lots of Parallelograms

15 min

Activity Narrative

While there is not one correct way to find the area of a parallelogram, each parallelogram here is designed to elicit a particular strategy. Monitor for students who use these two main strategies:

- Decomposing and rearranging the parallelograms into a rectangle. Parallelograms A and C encourage this strategy.
- Enclosing the parallelogram and subtracting the area of the extra pieces. Parallelogram B is not as easy to decompose and rearrange (though some students are likely to try that approach first) and may prompt this strategy.

The presence of a grid for Parallelograms A and B and its absence for Parallelogram C allow students to reason concretely and abstractly, respectively, about the measurements that they need to find the area (MP2).

With repeated reasoning, students may begin to see regularity in the segments and measurements that tend to be useful in finding area (MP8). Students formalize this awareness in an upcoming lesson. Highlighting the segments and measurements without referring to bases or heights can help support students' future work.

This is the first time that Math Language Routine 7: Compare and Connect is suggested in this course. In this routine, students are given a problem that can be approached using multiple strategies or representations, and they record their method for all to see. They then compare and identify correspondences across strategies by means of a teacher-led gallery walk with commentary or by teacher think-aloud (such as "I notice . . . I wonder . . ."). A typical discussion prompt is "What is the same and what is different?", comparing their own strategy to the strategies of others. The purpose of this routine is to allow students to make sense of mathematical strategies and, through constructive conversations, develop awareness of the language used as they compare, contrast, and connect other ways of thinking to their own.

Access for English Language Learners

This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Standards

Addressing **6.G.A.1**

Instructional Routines

- MLR7: Compare and Connect

Launch

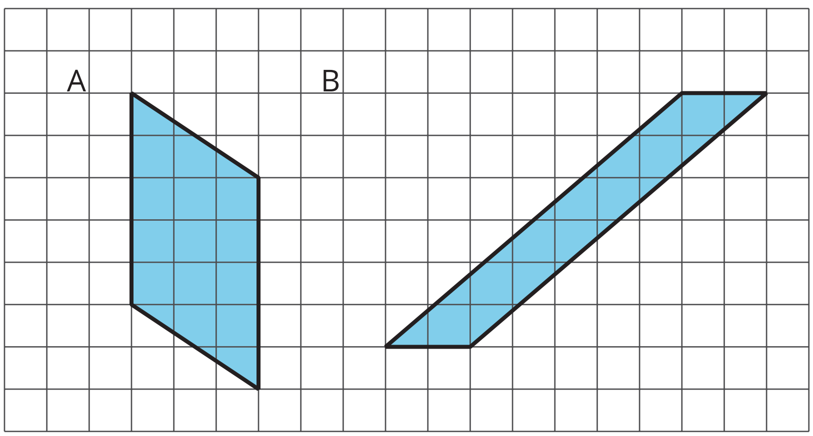
Keep students in groups of 2–4 and ask them to find the areas of several more parallelograms. Give them 5–7 minutes of quiet think time, followed by a couple of minutes to share their strategies with their groups. Ask students to attempt at least the first two questions individually before discussing with their group. Provide access to geometry toolkits.

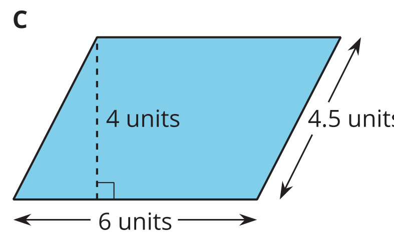
Encourage students to think, as they work through the problems, about which measurements of the parallelogram seem to be helpful for finding its area.

Select students who used each strategy described in the *Activity Narrative* to share later. Try to elicit both key mathematical ideas and a variety of student responses, especially from students who haven't shared recently.

Student Task Statement

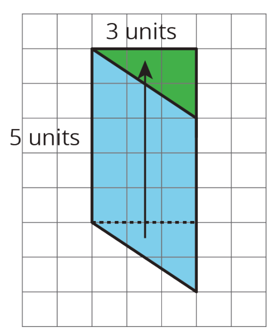
Find the area of each parallelogram. Show your reasoning.



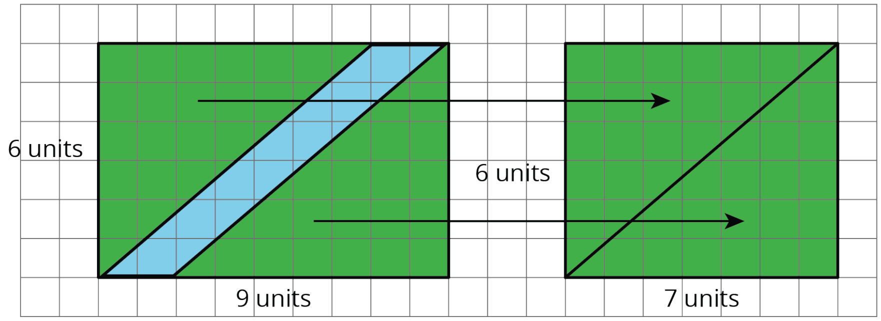


Student Response

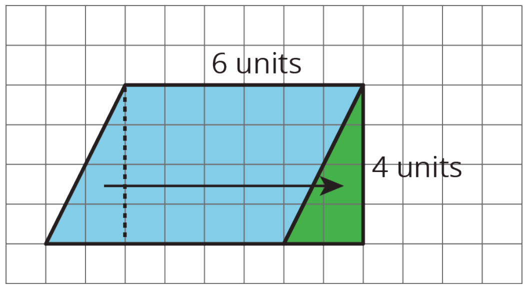
A: 15 square units. Sample reasoning: Decompose and rearrange the pieces to form a rectangle and multiply the base and side lengths of the rectangle to find the area. $5 \cdot 3 = 15$



B: 12 square units. Sample reasoning: Enclose the parallelogram with a rectangle and subtract the area of the extra pieces. The area of the extra pieces is found by rearranging the triangles to form a rectangle. $6 \cdot 9 - 6 \cdot 7 = 12$



C: 24 square units. Sample reasoning: Decompose the parallelogram and rearrange into a rectangle. Multiply the base and height lengths of the rectangle. $6 \cdot 4 = 24$



Building on Student Thinking

Some students may think that it is not possible to decompose and rearrange Parallelogram A because it has a pair of vertical sides instead of a pair of horizontal sides. Suggest that those students rotate their paper 90 degrees and back again to help them see that they could still use the same reasoning strategy regardless of the orientation. Also, they may find it helpful to first reason about area with the parallelogram rotated 90 degrees and then rotate it back to its original orientation.

Some students may spend time unsucessfully trying to decompose Parallelogram B into parts and rearrange the parts into a rectangle. Draw their attention to the list of strategies used in an earlier activity and urge them to try a different strateg**y.**

Activity Synthesis

The goal of this discussion is to highlight strategies and measurements that may be useful for finding the area of parallelograms with different characteristics or when given different information.

For each parallelogram, display 2 or 3 approaches used by previously selected students and share their strategies with everyone. If time allows, invite students to briefly describe their work. If an important strategy is not mentioned, bring it up and illustrate it. Use *Compare and Connect* to help students compare, contrast, and connect the different approaches. Ask students:

- "How are the strategies the same? How are they different?"
- "Did anyone solve the problem the same way, but would explain it differently?"

After hearing from students about all parallelograms, consider asking the following questions. Provide access to tracing paper or a copy of the drawing in case students wish to verify their thinking by physically cutting and rearranging pieces.

- "Which strategy—decomposing and rearranging, or enclosing and subtracting—seems more practical for finding the area of a shape similar to Parallelogram B? Why?" (Enclosing and subtracting, because it can be done in fewer steps. Decomposing the figure into small pieces could get confusing and lead to errors.)

- “If you decomposed Parallelogram C into a right triangle and another shape, how do you know that the cut-out piece actually fits on the other side, given that there's no grid to use?” (The two opposite sides of a parallelogram are parallel, so the longest side of the right triangle that is rearranged would match up perfectly with the segment on the other side.)
- “Three measurements are shown for Parallelogram C. Which ones did you use? Which ones did you not use? Why and why not?” (The 4 units and 6 units are side lengths of a rectangle that has the same area as that of the parallelogram. If we decompose the parallelogram with a vertical cut and move the piece on the left to the right to make a rectangle, the 4.5-unit length is no longer relevant.)

Lesson Synthesis

Revisit the definition of a parallelogram: A parallelogram has four sides. The opposite sides of a parallelogram are parallel.

Briefly revisit the last task, displaying for all to see the multiple strategies that students used. Point out that in some cases, students chose to decompose and rearrange parts, and in others they chose to enclose the parallelogram with a rectangle and subtract the area of the extra pieces from the area of the rectangle. Consider asking students: “What was it about each parallelogram that led you to choose a certain strategy?”

To help students make connections across strategies and generalize their observations, discuss questions such as:

- “When you decomposed and rearranged the parallelogram into another shape, did the area change?” (No.)
- “Why is it helpful to use a rectangle?” (We know how to find the area of a rectangle. We can multiply two adjacent side lengths—a base and a height.)
- “For those of you who enclosed the parallelogram with a rectangle, how did the two right triangles help you?” (They can be combined into a rectangle, whose area we can find and subtract from the area of the large rectangle.)
- “Which measurements or lengths were useful for finding the area of the parallelogram?" (One side length of the parallelogram and the length of a perpendicular segment between that side and the opposite side.) “Which lengths did you not use?” (The other side length.)

4.4 How Would You Find the Area?

Cool-down

5 min

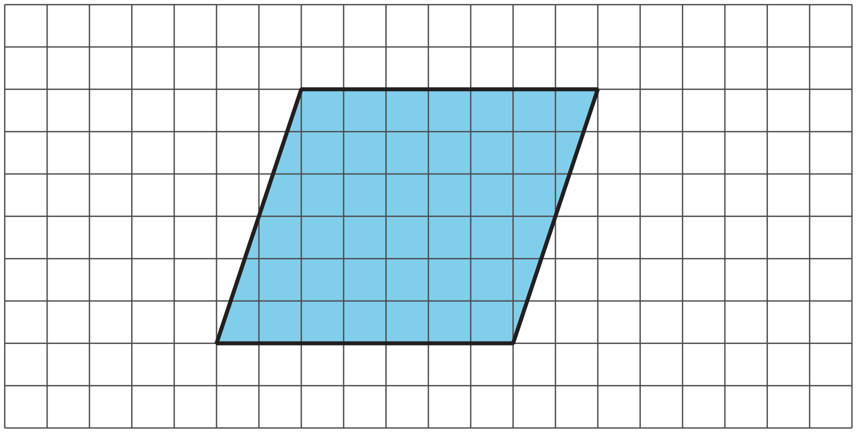
This activity sets the stage for the next lesson, which formalizes how to find the area of any parallelogram. Notice the strategies that students are currently using to help make connections to the algebraic expression $b \cdot h$ that they will see in the next lesson.

Standards

Addressing **6.G.A.1**

Student Task Statement

How would you find the area of this parallelogram? Describe your strategy.



Student Response

Sample responses:

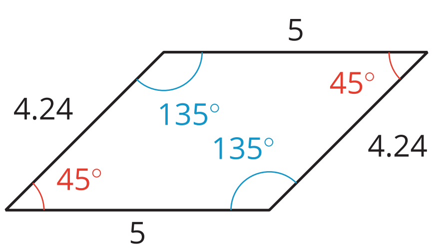
- Decompose a triangle from one side of the parallelogram and move it to the other side to make a rectangle. Multiply the base and side (height) lengths of the rectangle.
- Draw a rectangle that just fits around the parallelogram, multiply the bottom length of that rectangle by its side length to find the area of the rectangle, and then subtract the combined area of the triangles that do not belong to the parallelogram.
- Count how many squares are across the bottom of the parallelogram and how many squares tall it is and multiply them.

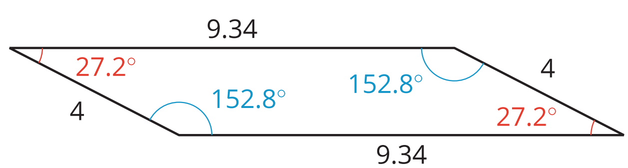
Responding to Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 4 Summary

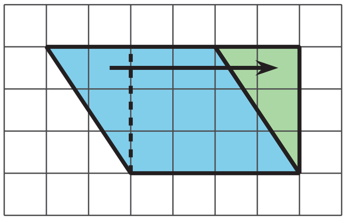
A *parallelogram* is a quadrilateral (it has four sides). The opposite sides of a parallelogram are parallel. The opposite sides of a parallelogram have the same length, and the opposite angles of a parallelogram have the same measure in degrees.

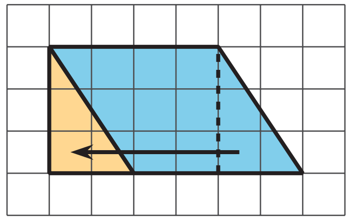


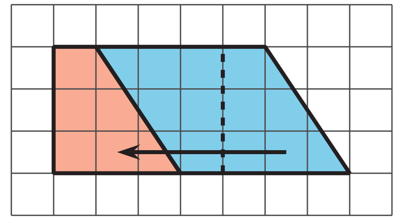


There are several strategies for finding the area of a parallelogram.

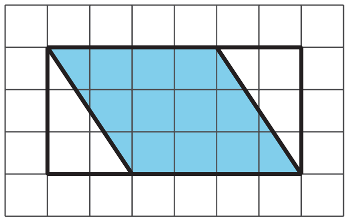
- We can decompose and rearrange a parallelogram to form a rectangle. Here are three ways:

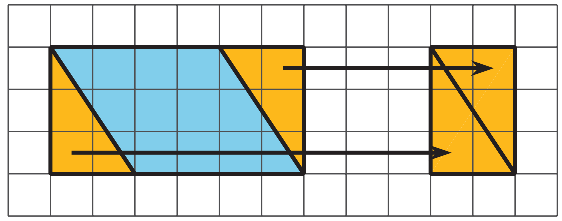




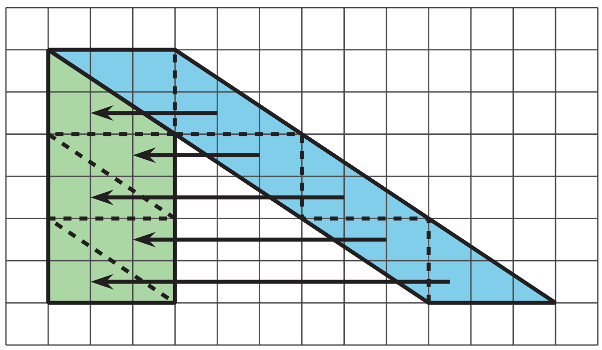


- We can enclose the parallelogram and then subtract the area of the two triangles in the corner.





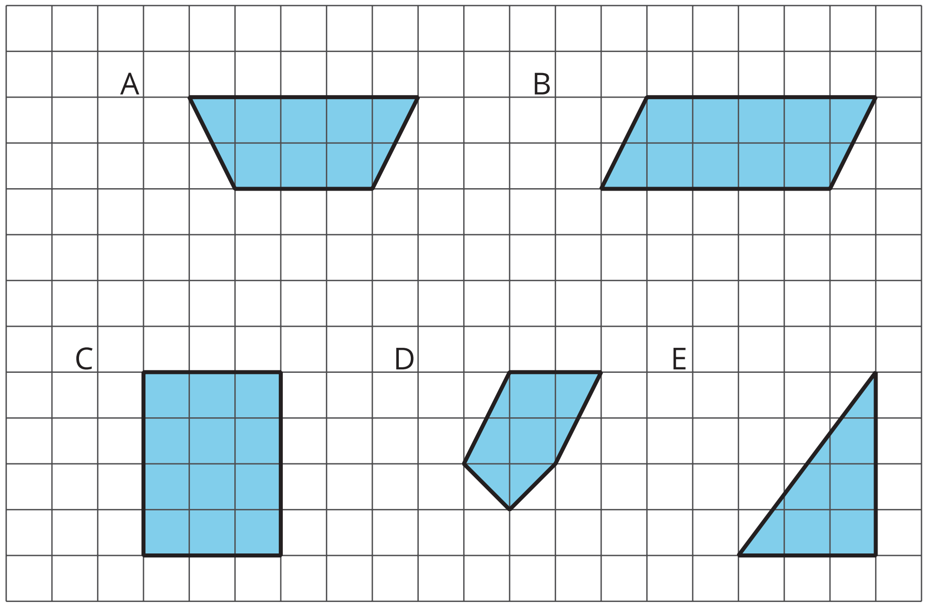
Both of these ways will work for any parallelogram. However, for some parallelograms the process of decomposing and rearranging requires a lot more steps than if we enclose the parallelogram with a rectangle and subtract the combined area of the two triangles in the corners.



Lesson 4 Practice Problems

1 Student Task Statement

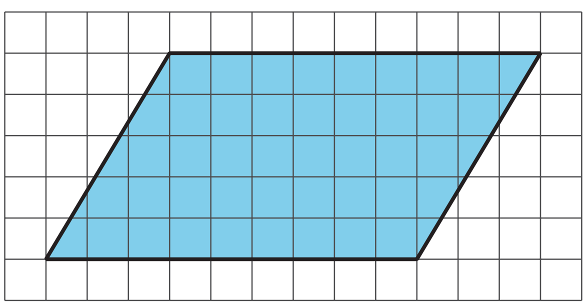
Select **all** of the parallelograms. For each figure that is *not* selected, explain how you know it is not a parallelogram.



Solution

B and C are parallelograms (C is also a rectangle). A is a trapezoid (two opposite sides are not parallel and two are not the same length), D is a pentagon, and E is a (right) triangle.

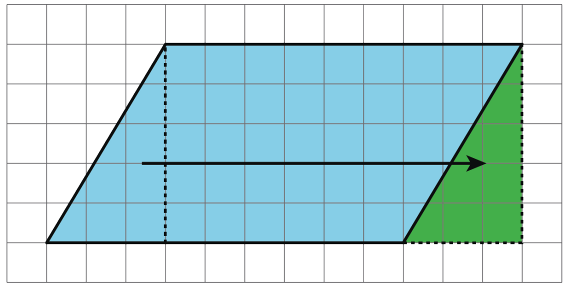
2 Student Task Statement



a. Decompose and rearrange this parallelogram to make a rectangle.
b. What is the area of the parallelogram? Explain your reasoning.

Solution

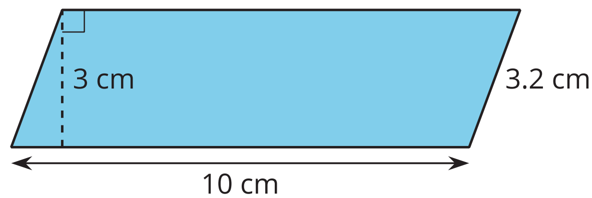
a. Sample response: The diagram shows that we get a rectangle that is 5 units by 9 units by decomposing and rearranging.



b. 45 square units. Sample reasoning: The area of the parallelogram is the same as the area of the rectangle, which is 45 square units.

3 Student Task Statement

Find the area of the parallelogram.

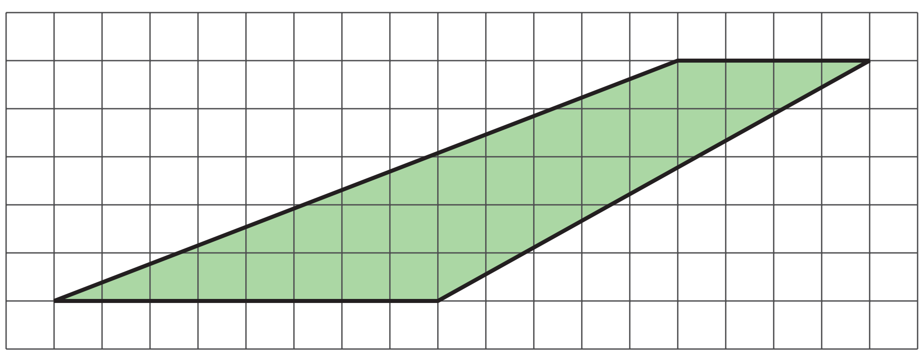


Solution

30 sq cm

4 Student Task Statement

Explain why this quadrilateral is *not* a parallelogram.



Solution

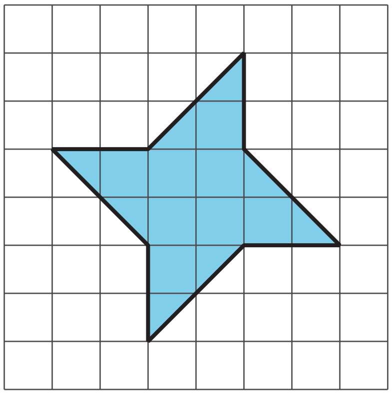
Sample response: Both pairs of opposite sides would need to be parallel and the same length, and that is not true. Also, the opposite angles are not equal.

5 from Unit 1, Lesson 3

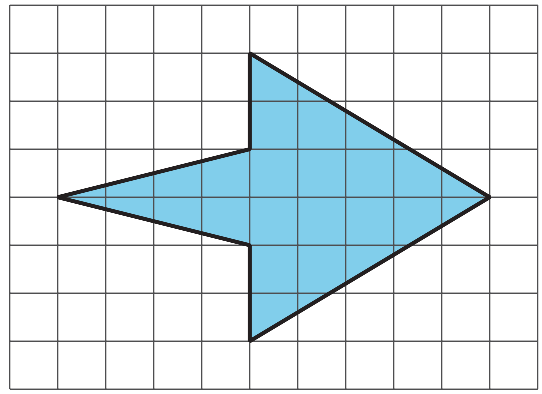
Student Task Statement

Find the area of each shape. Show your reasoning.

A



B



Solution

A: 12 square units. Sample reasoning: The shape can be decomposed into a square in the middle and 4 right triangles. The area of the square is 4 square units. Putting 2 triangles together also make a square with an area of 4 square units, so 4 triangles make 8 square unit. $4+8=12$

B: 19 square units. Sample reasoning: The shape can be decomposed into 2 triangles by using a vertical cut. The triangle on the left can be decomposed and rearranged into a rectangle that is 1-by-4 rectangle, which is 4 square units in area. The one on the right can be rearranged into a 3-by-5 rectangle, which is 15 square units in area. $4+15=19$