Some Numbers are Rational

$$0.4\overline{85}$$

Some Numbers are Rational

I want to turn this repeating decimal into a fraction. I can see this decimal number has a two-digit repeating pattern.

Some Numbers are Rational

$$x = 0.4\overline{85}$$

Some Numbers are Rational

First I'll set x equal to this number.

Some Numbers are Rational

$$100x = 48.5\overline{85}$$

Some Numbers are Rational

Since the repeating pattern is 2 digits long, I'm going to multiply by 100 and write out a few more digits so I can still see the pattern.

Some Numbers are Rational

$$\begin{array}{rr} 100x = & 48.5\overline{85} \\ -x = & -0.485\overline{85} \end{array}$$

Some Numbers are Rational

Now I'll subtract the value of the decimal from each side. By lining the subtraction up vertically, it's easier to see what the left side will equal.

Some Numbers are Rational

$$\begin{aligned} 99x &= 48.1 \\ 990x &= 481 \end{aligned}$$

Some Numbers are Rational

If I multiply each side by 10, I can re-write my equation without any decimal numbers.

Some Numbers are Rational

$$x = \frac{481}{990}$$

Some Numbers are Rational

Dividing each side by 990, I now know

$$0.4\overline{85} = \frac{481}{990}$$

CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.