Using Diagrams to Represent Addition and Subtraction

Let's represent addition and subtraction of decimals.

14.1 Do the Zeros Matter?

1. Find the value mentally: $1.009 + 0.391$

2. Decide if each statement is true or false. Be prepared to explain your reasoning.
   a. $34.56000 = 34.56$

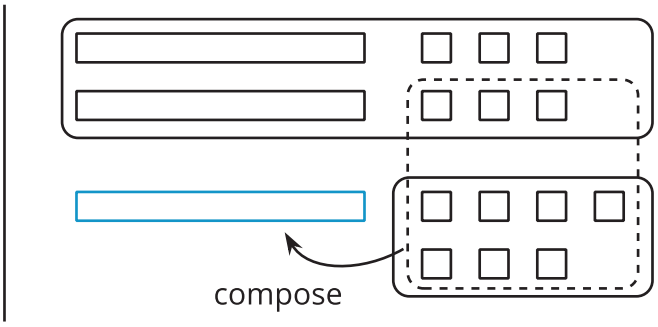
   b. $25 = 25.0$

   c. $2.405 = 2.45$

14.2 Finding Sums in Different Ways

1. Here are two ways to calculate the value of $0.26 + 0.07$. In the diagram, each rectangle represents 0.1 and each square represents 0.01.

$$\begin{array}{r} {\scriptstyle 1} \\ 0.26 \\ +\ 0.07 \\ \hline 0.33 \end{array}$$



Discuss with your partner:

   a. Why can 10 ten squares be composed into a rectangle?

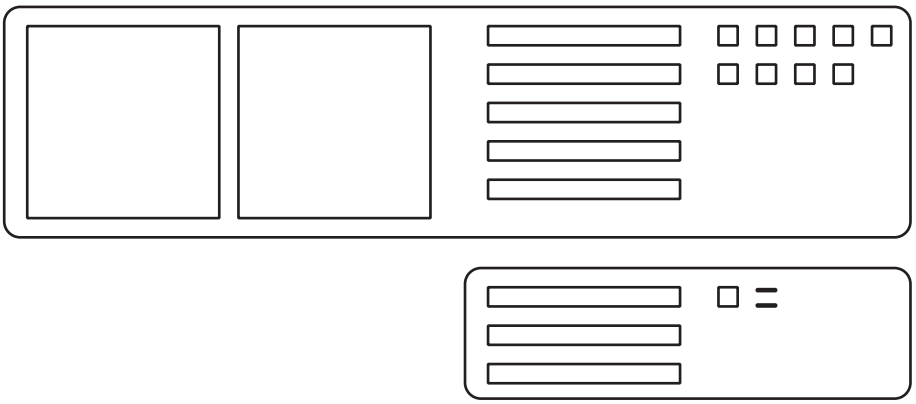
   b. How is this composition represented in the vertical calculation?

2. Find the value of $0.38 + 0.69$ by using base-ten blocks or a diagram. Can you find the sum without composing a larger unit? Would it be useful to compose some pieces? Be prepared to explain your reasoning.

3. Calculate $0.38 + 0.69$. Check if the sum is the same as the value of the base-ten blocks or diagram you used earlier.

4. Find each sum. The larger square represents 1. The rectangle represents 0.1. The small square represents 0.01.

   a.



   b.

$$\begin{array}{r} 6.03 \\ +\ 0.098 \\ \hline \end{array}$$

Are you ready for more?

In a game, special stones are used for bartering. The values of the stones are based on their color and are ranked as shown, with red having the highest value.

red
orange
yellow
green
blue
indigo
violet

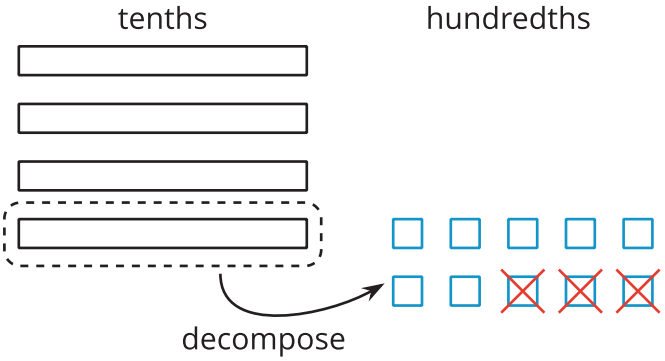
Each color is valued at 3 times the color below it in the ranking. So the value of a red stone is 3 times that of an orange stone, and the value of a green stone is 3 times that of a blue stone.

If you had 500 violet stones and wanted to trade so that you would carry as few stones as possible, which stones would you have? Explain or show your reasoning.

14.3 Subtracting Decimals of Different Lengths

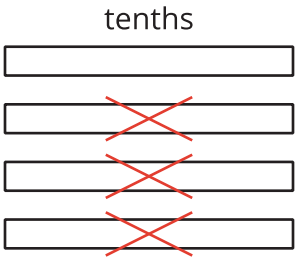
Diego and Noah drew different diagrams to represent $0.4 - 0.03$. Each rectangle represents 0.1. Each square represents 0.01.

- Diego started by drawing 4 rectangles to represent 0.4. He then replaced 1 rectangle with 10 squares and crossed out 3 squares to represent subtraction of 0.03, leaving 3 rectangles and 7 squares in his diagram.



Diego's Method

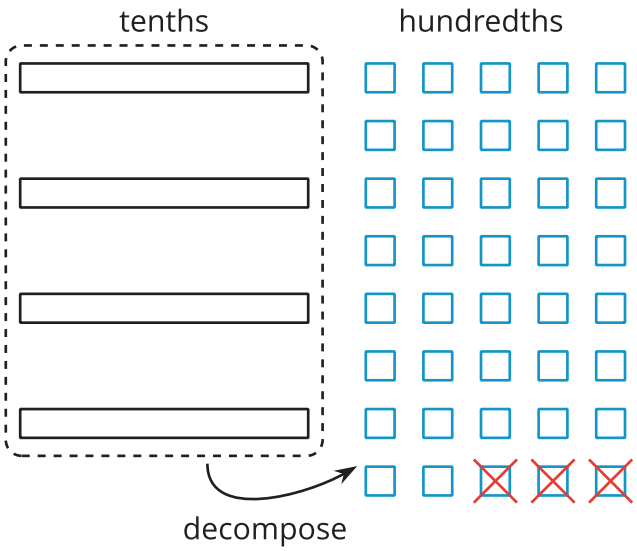
- Noah started by drawing 4 rectangles to represent 0.4. He then crossed out 3 rectangles to represent the subtraction, leaving 1 rectangle in his diagram.



Noah's Method

1. Do you agree that either diagram correctly represents $0.4 - 0.03$? Discuss your reasoning with a partner.

2. Elena also drew a diagram to represent $0.4 - 0.03$. She started by drawing 4 rectangles. She then replaced all 4 rectangles with 40 squares and crossed out 3 squares to represent subtraction of 0.03, leaving 37 squares in her diagram. Is her diagram correct? Discuss your reasoning with a partner.



Elena's Method

3. Find each difference. Be prepared to explain your reasoning. If you get stuck, you can use base-ten blocks or diagrams to represent each expression and find its value.

   a. $0.3 - 0.05$

   b. $2.1 - 0.4$

c. $1.03 - 0.06$

d. $0.02 - 0.007$

Are you ready for more?

In a game, special stones are used for bartering. The values of the stones are based on their color and are ranked as shown, with red having the highest value.

red
orange
yellow
green
blue
indigo
violet

Each color is valued at 3 times the color below it in the ranking. So the value of a red stone is 3 times that of an orange stone, and the value of a green stone is 3 times that of a blue stone.

The stones can be used to buy items. Suppose you want to buy a tool that is worth 2 yellow stones, 2 green stones, 2 blue stones, and 1 indigo stone. You go into the store with 1 red stone, 1 yellow stone, 2 green stones, 1 blue stone, and 2 violet stones.

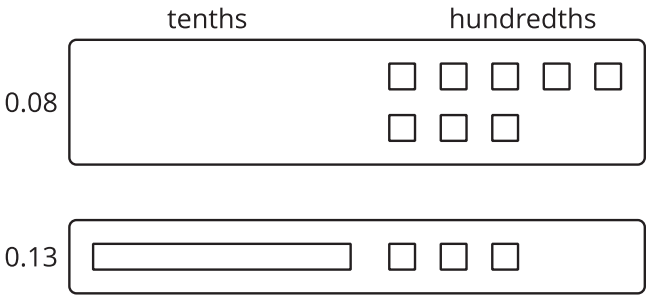
What stones would the shopkeeper give you for the change? Assume the shopkeeper would use as few stones as possible.

Lesson 14 Summary

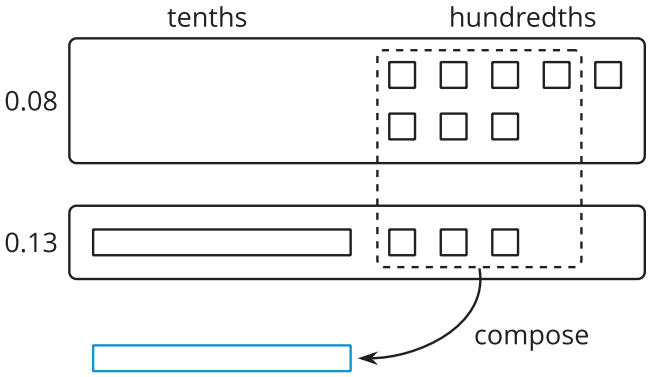
Base-ten diagrams represent collections of base-ten units—tens, ones, tenths, hundredths, and so on. We can use them to help us understand sums of decimals.

Suppose we are finding $0.08 + 0.13$.

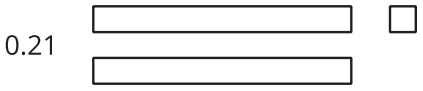
Here is a diagram where a square represents 0.01 and a rectangle (made up of ten squares) represents 0.1.



To find the sum, we can compose 10 hundredths into 1 tenth.



We now have 2 tenths and 1 hundredth, so $0.08 + 0.13 = 0.21$.



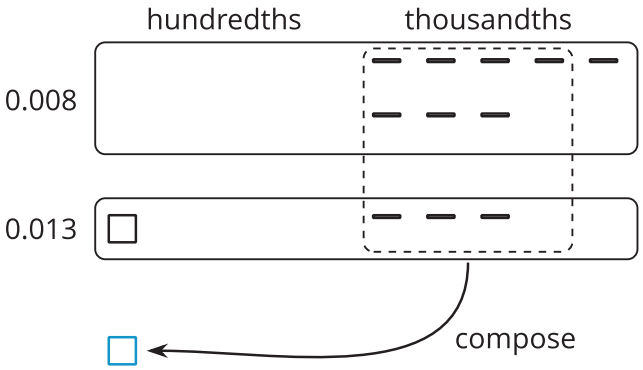
We can also use vertical calculation to find $0.08 + 0.13$.

Notice how this representation also shows 10 hundredths are composed into 1 tenth.

$$\begin{array}{r} {\scriptstyle 1} \\ 0.13 \\ +\ 0.08 \\ \hline 0.21 \end{array}$$

This works for any decimal place.

Suppose we are finding $0.08 + 0.13$. Here is a diagram in which a small rectangle represents 0.001. We can compose 10 thousandths into 1 hundredth.



The sum is 2 hundredths and 1 thousandth.

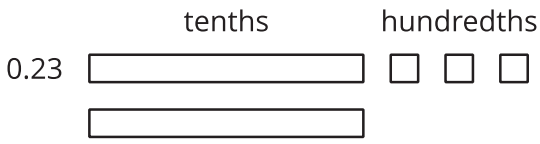


Here is a vertical calculation of $0.008 + 0.013$.

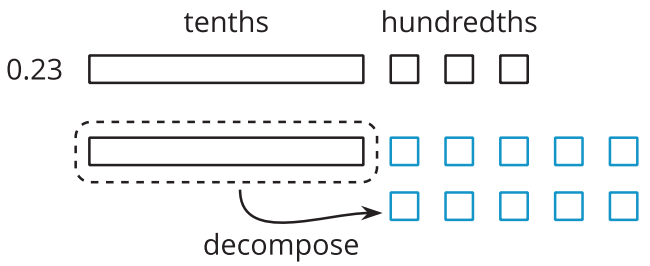
$$\begin{array}{r} {\scriptstyle 1} \\ 0.013 \\ +\ 0.008 \\ \hline 0.021 \end{array}$$

Base-ten diagrams can also help us understand subtraction.

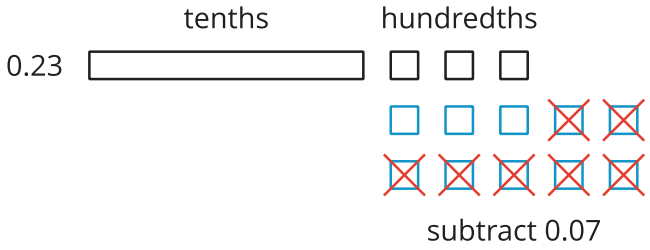
Suppose we are finding $0.23 - 0.07$. Here is a diagram showing 0.23, or 2 tenths and 3 hundredths.



Subtracting 7 hundredths means removing 7 small squares, but we do not have enough to remove. Because 1 tenth is equal to 10 hundredths, we can decompose one of the tenths (1 rectangle) into 10 hundredths (10 small squares).



We now have 1 tenth and 13 hundredths, from which we can remove 7 hundredths.



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We have 1 tenth and 6 hundredths remaining, so $0.23 - 0.07 = 0.16$.



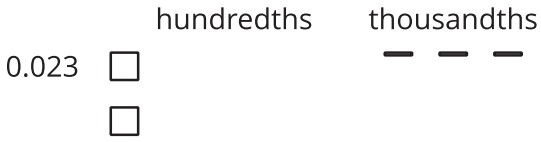
Here is a vertical calculation of $0.23 - 0.07$.

Notice how this representation also shows that a tenth is decomposed into 10 hundredths in order to subtract 7 hundredths.

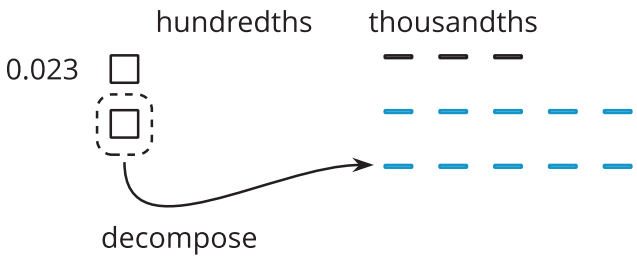
$$\begin{array}{r} {\scriptstyle 1\ 13} \\ 0.2\cancel{3} \\ -\ 0.0\ 7 \\ \hline 0.1\ 6 \end{array}$$

This works for any decimal place.

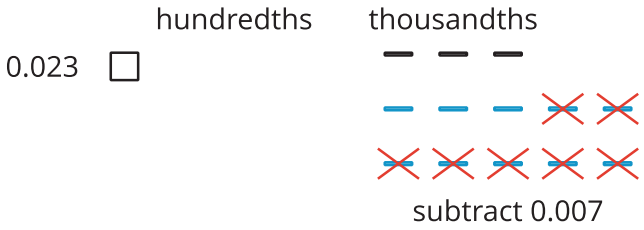
Suppose we are finding $0.023 - 0.007$. Here is a diagram showing 0.023.



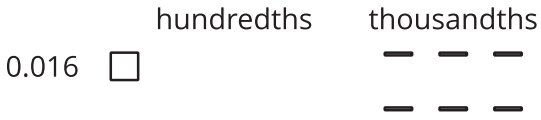
We want to remove 7 thousandths (7 small rectangles). We can decompose one of the hundredths into 10 thousandths.



Now we can remove 7 thousandths.



We have 1 hundredth and 6 thousandths remaining, so $0.023 - 0.007 = 0.016$.



Here is a vertical calculation of $0.023 - 0.007$.

$$\begin{array}{r} {\scriptstyle 1\ 13} \\ 0.0\cancel{2}\cancel{3} \\ -\ 0.0\ 0\ 7 \\ \hline 0.0\ 1\ 6 \end{array}$$

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