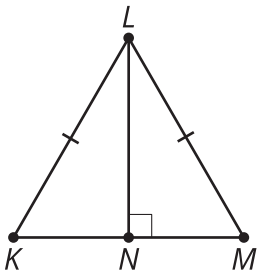
Other Conditions for Triangle Similarity

Let's prove that more triangles are similar.

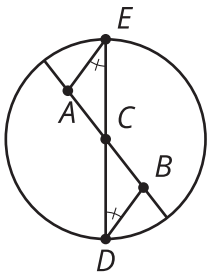
10.1 Triangle Congruence

Is there enough information to determine if the pairs of triangles are congruent? If so, what theorem(s) would you use? If not, what additional piece of information could you use?

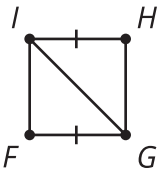
- Triangle LMN and triangle LKN $\overline{KM} \perp \overline{NL}, \overline{KL} \cong \overline{ML}$



- Triangle ACE and triangle BCD $\angle E \cong \angle D$



- Triangle GHI and triangle IFG $\overline{HI} \cong \overline{FG}$



10.2 Side-Angle-Side Triangle Similarity?

Andre remembers lots of ways to prove that triangles are congruent. He asks Clare, "Can we use Angle-Side-Angle to prove that triangles are similar?"

Clare: "Sure, but we don't need the Side part because Angle-Angle is enough to prove that triangles are similar."

Andre: "Hmm, what about Side-Angle-Side or Side-Side-Side? What if we don't know 2 angles?"

Clare: "Oh! I don't know. Let's draw a picture and see if we can prove it."

Andre: "Uh-oh. If 'side' means corresponding sides with the same length, then we'll only get congruent triangles."

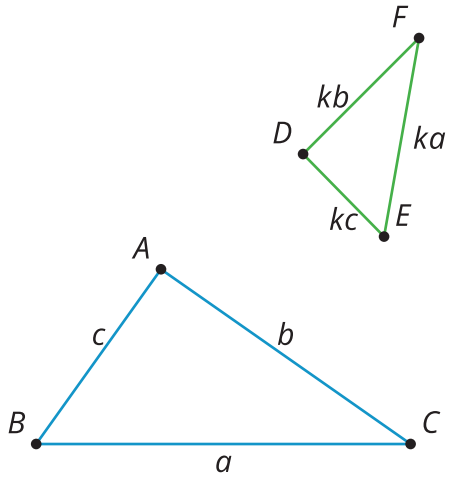
1. What could "side" stand for to prove triangles similar?

2. Draw a diagram that would help you prove the Side-Angle-Side Triangle Similarity Theorem.

3. Write a proof.

10.3 Side-Side-Side Triangle Similarity

Prove that these 2 triangles must be similar.



Are you ready for more?

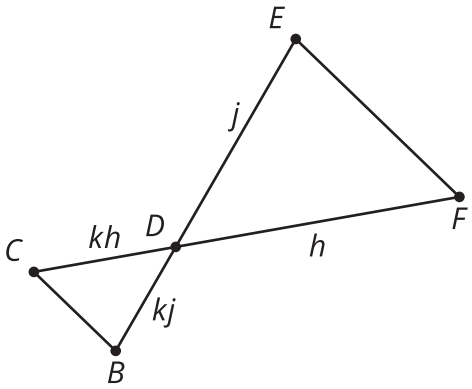
Prove or disprove the Side-Side-Angle Triangle Similarity Theorem.

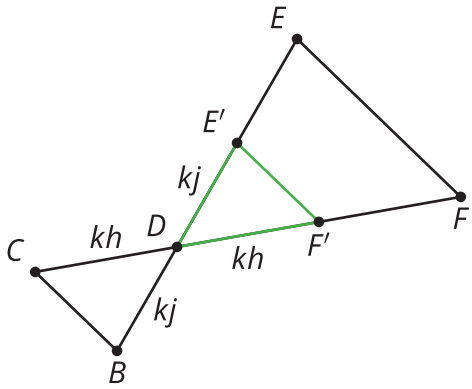
Lesson 10 Summary

Besides the Angle-Angle Triangle Similarity Theorem, what other criteria are sufficient to prove triangles similar?

When two sides of one triangle are proportional to two corresponding sides of a second triangle, using the same scale factor, k, and the pair of angles between these sides are congruent, then the triangles are similar by the Side-Angle-Side Triangle Similarity Theorem.

For example, angles EDF and BDC are vertical angles and so they are congruent, and there are two pairs of corresponding sides with the same scale factor.





Dilate triangle DEF using center D and a scale factor of k. Because $\frac{BD}{ED} = \frac{CD}{FD} = k$, BD is now congruent to $E'D$, and CD is congruent to $F'D$. The dilation did not change the size of the angles. Therefore, triangle $E'DF'$ is congruent to triangle BDC by the Side-Angle-Side Triangle Congruence Theorem. This means that there is a sequence of rigid motions that takes triangle $E'DF'$ to triangle BDC. That means that triangle BDC is similar to triangle EDF because there is a dilation and a sequence of rigid motions that takes one to the other. There wasn't anything special about these two triangles. Therefore, any pair of triangles with two pairs of sides whose lengths are in the same proportion and with the angle between them congruent must be similar.

We can also show that if all three pairs of corresponding sides are proportional and use the same scale factor, k, this is sufficient to prove that the triangles are similar. We call this the Side-Side-Side Triangle Similarity Theorem.