## Lesson 13: Amplitude and Midline

* Let's transform the graphs of trigonometric functions

### 13.1: Comparing Parabolas

Match each equation to its graph.

1. $y=x^{2}$
2. $y=3x^{2}$
3. $y=3\left(x−1\right)^{2}$
4. $y=3x^{2}−1$
5. $y=x^{2}−1$

A



B



C



D



E



Be prepared to explain how you know which graph belongs with each equation.

### 13.2: Blowing in the Wind



Suppose a windmill has a radius of 1 meter and the center of the windmill is $\left(0,0\right)$ on a coordinate grid.

1. Write a function describing the relationship between the height $h$ of $W$ and the angle of rotation $θ$. Explain your reasoning.
2. Describe how your function and its graph would change if:
	1. the windmill blade has length 3 meters.
	2. The windmill blade has length 0.5 meter.
3. Test your predictions using graphing technology.

### 13.3: Up, Up, and Away

1. A windmill has radius 1 meter and its center is 8 meters off the ground. The point $W$ starts at the tip of a blade in the position farthest to the right and rotates counterclockwise. Write a function describing the relationship between the height $h$ of $W$, in meters, and the angle $θ$ of rotation.
2. Graph your function using technology. How does it compare to the graph where the center of windmill is at $\left(0,0\right)$?
3. What would the graph look like if the center of the windmill were 11 meters off the ground? Explain how you know.

#### Are you ready for more?

Here is the graph of a different function describing the relationship between the height $y$, in feet, of the tip of a blade and the angle of rotation $θ$ made by the blade. Describe the windmill.



### Lesson 13 Summary

Suppose a bike wheel has radius 1 foot and we want to determine the height of a point $P$ on the wheel as it spins in a counterclockwise direction. The height $h$ in feet of the point $P$ can be modeled by the equation $h=sin\left(θ\right)+1$ where $θ$ is the angle of rotation of the wheel. As the wheel spins in a counterclockwise direction, the point first reaches a maximum height of 2 feet when it is at the top of the wheel, and then a minimum height of 0 feet when it is at the bottom.



The graph of the height of $P$ looks just like the graph of the sine function but it has been raised by 1 unit:



The horizontal line $h=1$, shown here as a dashed line, is called the **midline** of the graph.

What if the wheel had a radius of 11 inches instead? How would that affect the height $h$, in inches, of point $P$ over time? This wheel can also be modeled by a sine function, $h=11sin\left(θ\right)+11$, where $θ$ is the angle of rotation of the wheel. The graph of this function has the same wavelike shape as the sine function but its midline is at $h=11$ and its **amplitude** is different:



The amplitude of the function is the length from the midline to the maximum value, shown here with a dashed line, or, since they are the same, the length from the minimum value to the midline. For the graph of , the midline value is 11 and the maximum is 22. This means the amplitude is 11 since $22−11=11$.



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