Solving Quadratics

Let's solve quadratic equations.

16.1 Find the Perfect Squares

The expression $x^2 + 8x + 16$ is equivalent to $(x + 4)^2$. Which expressions are equivalent to $(x + n)^2$ for some number n?

1. $x^2 + 10x + 25$

2. $x^2 + 10x + 29$

3. $x^2 - 6x + 8$

4. $x^2 - 6x + 9$

16.2 Different Ways to Solve It

Elena and Han solved the equation $x^2 - 6x + 7 = 0$ in different ways.

Elena said, "First I added 2 to each side:	$x^2 - 6x + 7 + 2 = 2$
So that tells me:	$(x - 3)^2 = 2$
I can find the square roots of both sides:	$x - 3 = \pm\sqrt{2}$
Which is the same as:	$x = 3 \pm \sqrt{2}$
So the two solutions are	$x = 3 + \sqrt{2}$ and $x = 3 - \sqrt{2}$."

Han said, "I used the quadratic formula:	$x = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$
Since $x^2 - 6x + 7 = 0$, that means $a = 1$, $b =$ -6, and $c = 7$. I know:	$x = \frac{6 \pm \sqrt{36 - 4 \cdot 1 \cdot 7}}{2 \cdot 1}$
or	$x = \frac{6 \pm \sqrt{8}}{2}$
So:	$x = 3 \pm \frac{\sqrt{8}}{2}$
I think the solutions are	$x = 3 + \frac{\sqrt{8}}{2}$ and $x = 3 - \frac{\sqrt{8}}{2}$."

Do you agree with either of them? Explain your reasoning.

Are you ready for more?

Under what circumstances would solving an equation of the form $x^2 + bx + c = 0$ lead to a solution that doesn't involve fractions?

16.3 Solve These Ones

Solve each quadratic equation with a method of your choice. Be prepared to compare your approach with a partner's.

1. $x^2 = 100$
2. $x^2 = 38$
3. $x^2 - 10x + 25 = 0$
4. $x^2 + 14x + 40 = 0$
5. $x^2 + 14x + 39 = 0$
6. $3x^2 - 5x - 11 = 0$

Lesson 16 Summary

Consider the quadratic equation $x^2 - 5x = 25$. One way to solve equations like this is by completing the square.

To complete the square, note that the perfect square $(x+n)^2$ is equal to $x^2 + (2n)x + n^2$. Compare the coefficients of x in $x^2 + (2n)x + n^2$ to our expression $x^2 - 5x$ to see that we want $2n = \text{-}5$, or just $n = \text{-}\frac{5}{2}$.

This means the perfect square $\left(x - \frac{5}{2}\right)^2$ is equal to $x^2 - 5x + \frac{25}{4}$, so adding $\frac{25}{4}$ to each side of our equation will give us a perfect square.

$$x^2 - 5x = 25$$
$$x^2 - 5x + \frac{25}{4} = 25 + \frac{25}{4}$$
$$\left(x - \frac{5}{2}\right)^2 = \frac{100}{4} + \frac{25}{4}$$
$$\left(x - \frac{5}{2}\right)^2 = \frac{125}{4}$$

The two numbers that square to make $\frac{125}{4}$ are $\frac{\sqrt{125}}{2}$ and $\text{-}\frac{\sqrt{125}}{2}$, so: $x - \frac{5}{2} = \pm\frac{\sqrt{125}}{2}$

which means the two solutions are: $x = \frac{5}{2} \pm \frac{\sqrt{125}}{2}$

Now let's look at another quadratic equation $3x^2 - 2x = 0.8$.

We could divide each side by 3 and then complete the square like before, but let's use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To use this formula, we first need to put the equation in standard form and identify a, b, and c. Rearranging, we get:

$$3x^2 - 2x - 0.8 = 0$$

so $a = 3$, $b = \text{-}2$, and $c = \text{-}0.8$. We have to be careful to pay attention to the negative signs. Using the quadratic formula, we get:

$$x = \frac{\text{-}(\text{-}2) \pm \sqrt{(\text{-}2)^2 - 4(3)(\text{-}0.8)}}{2(3)}$$
$$x = \frac{2 \pm \sqrt{4 + (12)(0.8)}}{6}$$

Evaluating these solutions with a calculator gives decimal approximations -0.281 and 0.948.

