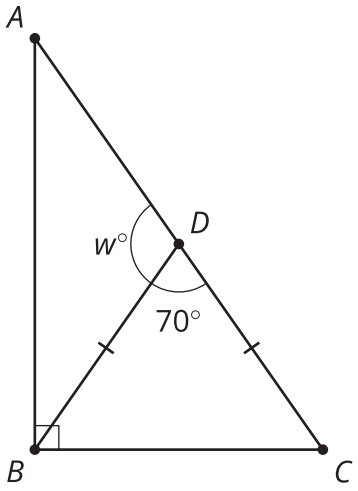
Unknown Angles

Let's find the measures of unknown angles.

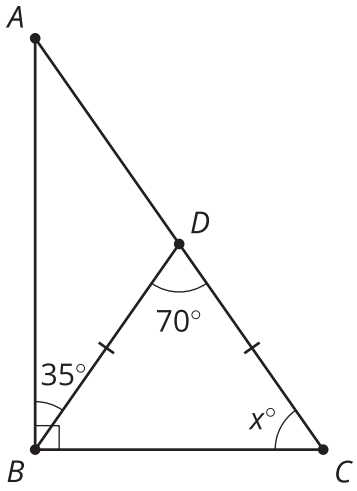
5.1 Math Talk: Unknown Angles

Find the value of each variable mentally.

- $\overline{BD} \cong \overline{CD}$



- $\overline{BD} \cong \overline{CD}$



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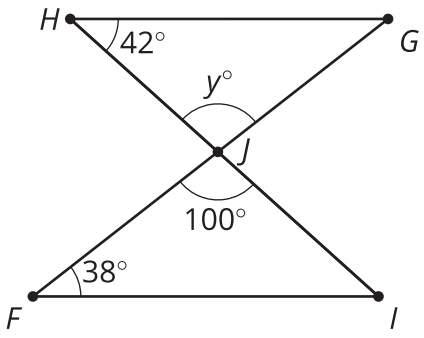

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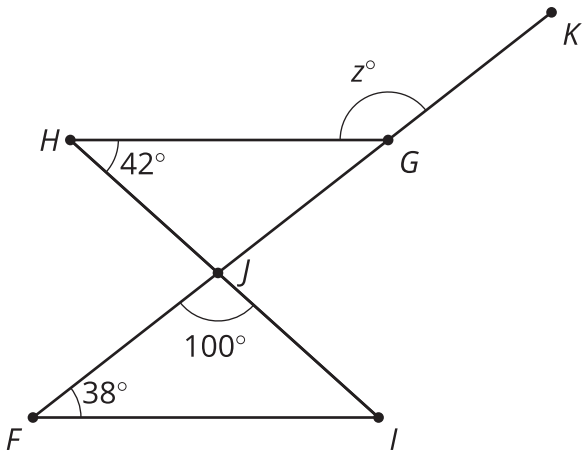
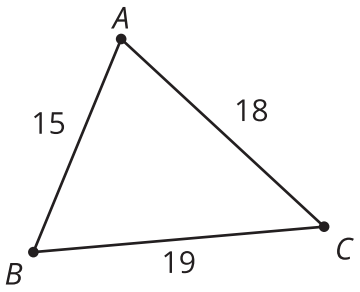


5.2 More Unknown Angles

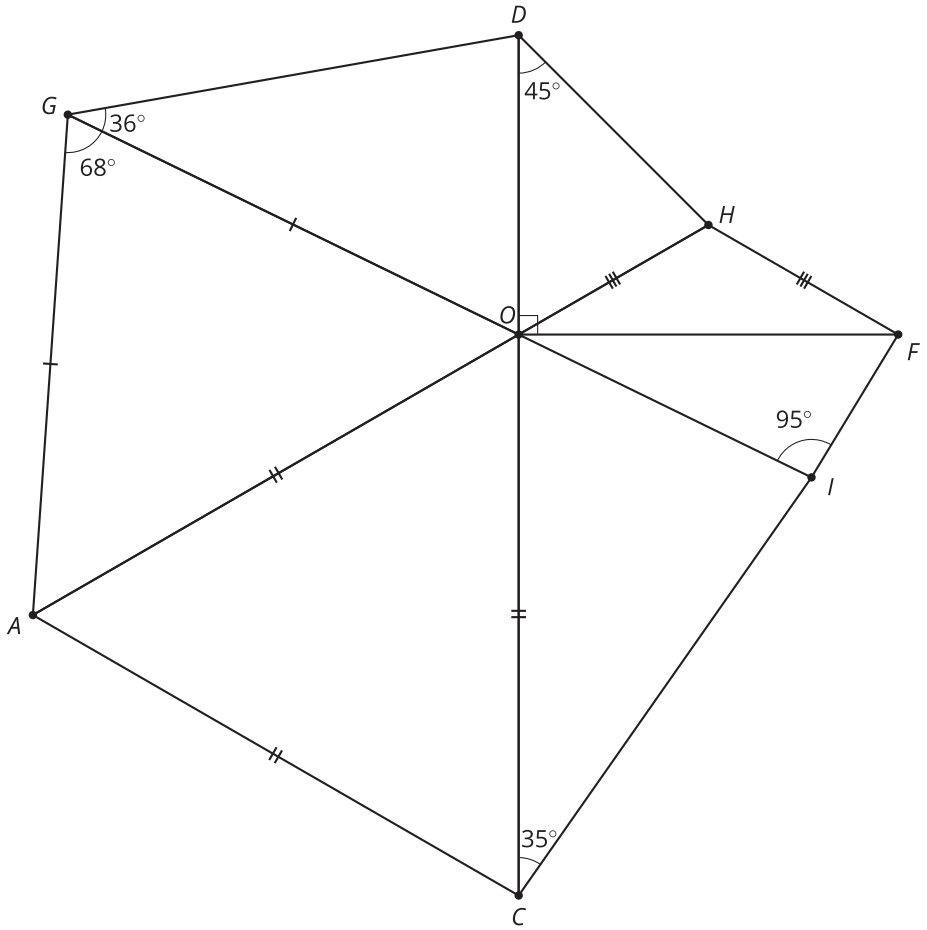
1. Here is triangle ABC. List the angles in order from smallest to largest.



2. In triangle XYZ (not drawn), side XZ is the longest. Which angle is the largest? Explain how you know.

5.3 The Wacky Pinwheel

$\overline{GO} \cong \overline{GA}, \overline{CO} \cong \overline{AO} \cong \overline{AC}, \overline{HO} \cong \overline{HF}, \overline{DO} \perp \overline{OF}$



1. Given that triangle GOA and triangle FOH are isosceles, triangle AOC is equilateral, and the measure of angle DOF is 90 degrees, label all unknown angle measures in the figure.

2. For each triangle listed in the table, name the longest side and shortest side.

triangle	longest side(s)	shortest side(s)
AOC		
GOA		
DOG		
HOD		
FOH		
IOF		
COI		

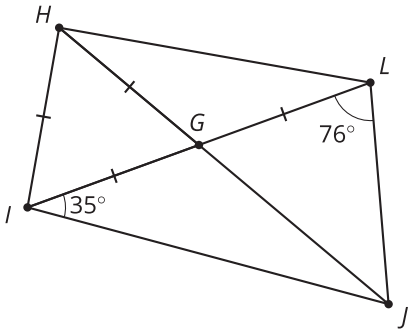
Are you ready for more?

Is it possible to identify the longest side in the figure? What about the shortest side?

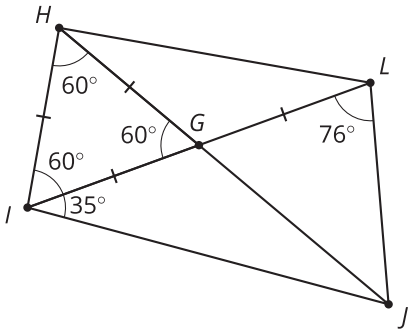
Lesson 5 Summary

Suppose you need to find the unknown angles in a complicated figure and figure out which side is the longest. Here is a figure with several unknown angles.

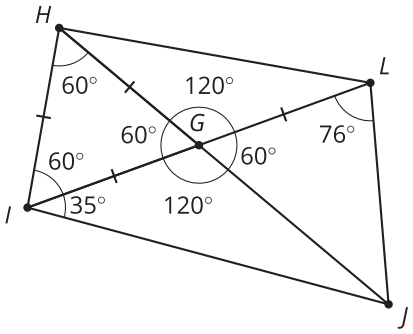
$\overline{HI} \cong \overline{HG} \cong \overline{IG} \cong \overline{GL}$



One way to start is to consider the equilateral triangle HGI. Since all sides are congruent, all angles in that triangle are congruent. Since the angles must add up to 180 degrees, each angle in triangle HGI is 60 degrees, because $\frac{180}{3} = 60$.

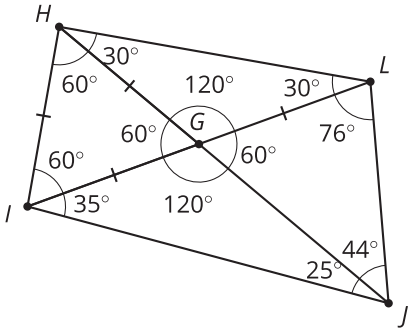


Next, consider the angles with a vertex at G. Using rules for supplementary and vertical angles, we can find these unknown angles. Angle HGL is supplementary to angle HGI, so the measure of angle HGL is 120 degrees because $180 - 60 = 120$. Since angle LGJ forms a vertical pair with angle HGI, these angles are congruent. Similarly, angle HGL forms a vertical pair with angle IGJ, so the measures of angles HGL and IGJ are equal.



Triangle HGL is isosceles, so we can use the Isosceles Triangle Theorem to find the measure of the two base angles. The measures of angles GHI and GLH are both 30 degrees because $\frac{180-120}{2} = 30$.

Finally, we can use the Triangle Angle Sum Theorem to find the two unknown angles. The measure of angle LJG is 45 degrees, because $180 - (76 + 60) = 44$, and the measure of angle GJI is 25 degrees, because $180 - (120 + 35) = 25$.



With this knowledge, we can say that HL is the longest side in triangle HGL and IJ is the longest side in triangle IGJ. This is not enough information to know which side is the longest overall or help us find the exact lengths of any of the sides.