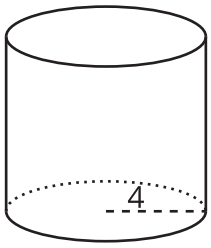
Volume

Let's calculate volumes of prisms, cylinders, cones, and spheres.

10.1 Different Bases

Here are some solids. Find the area of each base.

1



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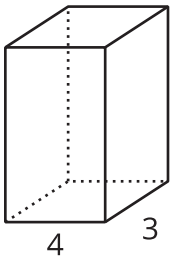

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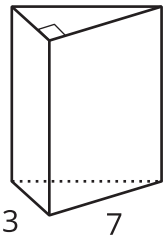

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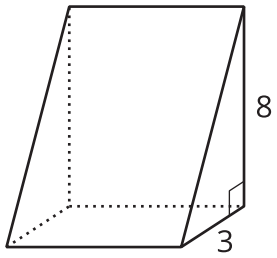
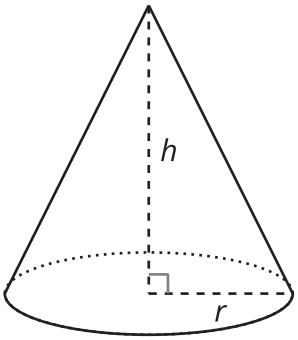
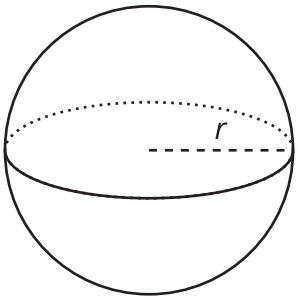


10.2 Circular Solids

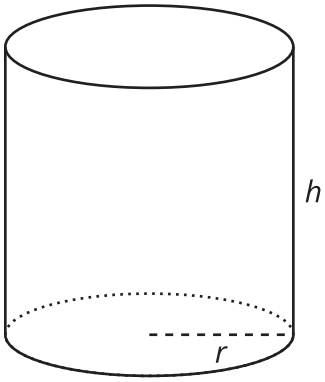
Here are some volume formulas:



$V = \frac{1}{3}\pi r^2 h$



$V = \frac{4}{3}\pi r^3$



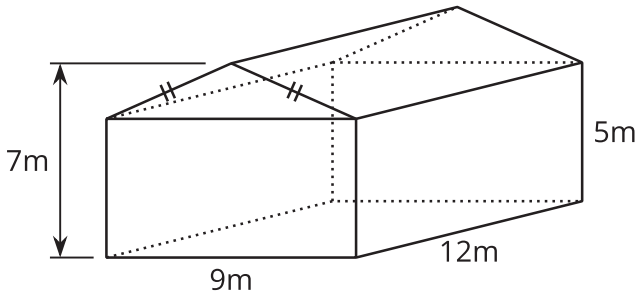
$V = \pi r^2 h$

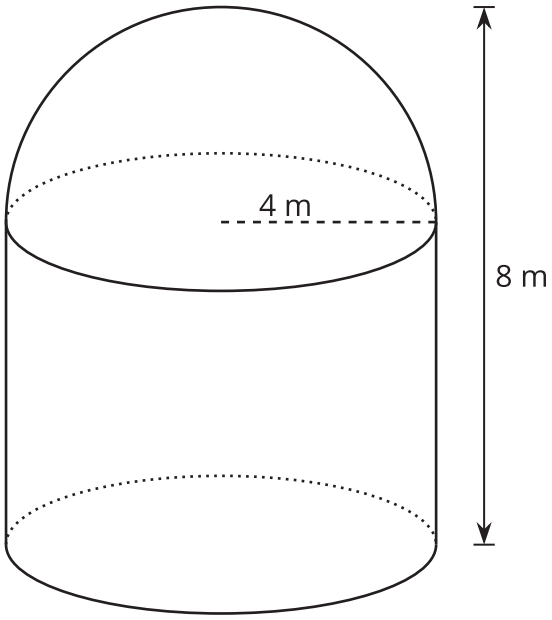
1. If a cone and cylinder have the same radius and the same height, which has the greater volume? Explain or show your reasoning.

2. Suppose you are given a cone, sphere, and cylinder. Find the volume of each solid if the heights of the cone and cylinder are 5 units and all three solids have a radius of 3 units.

3. Suppose you are given another cone, sphere, and cylinder. Find the radius of each figure if the volume is 128π cubic units and the height of the cone and cylinder is 12 units.

10.3 Storage Space





A farmer wants to know the sizes of the inside of her barn and silo. Here is the information she has:

- The inside of the barn is 12 meters long and 9 meters wide, two of its walls have a height of 5 meters, and the other two walls measure 7 meters from base to peak.
- The inside of the silo has a radius of 4 meters. It is capped with a hemispherical roof and measures 8 meters from the floor to the top of the dome.

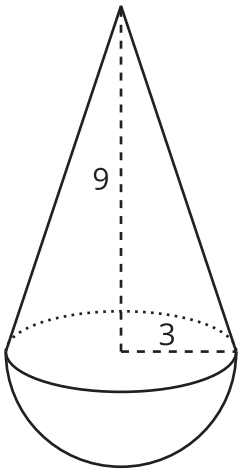
Find the total volume of:

1. The barn.
2. The silo.

Explain or show your reasoning.

Lesson 10 Summary

We can find the volume of a figure composed of three-dimensional solids by decomposing it into separate familiar shapes, finding the volume of each shape, and adding those volumes together.



For example, consider the composite figure shown. To calculate its volume, we start by decomposing it into a hemisphere and a cone. Then we find the volume of each shape.

We can find the volume of the cone using the formula $V = \frac{1}{3}\pi r^2 h$. The cone's radius measures 3 units, and its height measures 9 units, so its volume is 27π cubic units because $\frac{1}{3}\pi(3)^2 \cdot 9 = 27\pi$.

We can find the volume of a sphere using the formula $V = \frac{4}{3}\pi r^3$. To find the volume of a hemisphere, we take half the volume of a sphere, which is $\frac{1}{2} \cdot \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$. The volume of this hemisphere is 18π cubic units because $\frac{2}{3}\pi(3)^3 = 18\pi$.

Lastly, we add the volume of the cone and hemisphere to get the volume of the solid: 45π cubic units, because $27\pi + 18\pi = 45\pi$.