

# Unknown Exponents

Let's find unknown exponents.

## 8.1 A Bunch of $x$ 's

Solve each equation. Be prepared to explain your reasoning.

1.  $\frac{x}{3} = 12$

2.  $3x^2 = 12$

3.  $x^3 = 12$

4.  $\sqrt[3]{x} = 12$

5.  $\sqrt{3x} = 12$

6.  $\frac{3}{x} = 12$

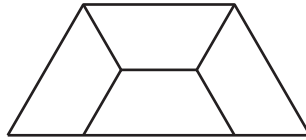
## 8.2

## A Tessellated Trapezoid

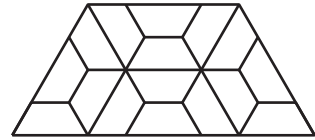
Here is a pattern showing a trapezoid decomposed into similar trapezoids at each step.



Step 0



Step 1



Step 2

1. If  $n$  is the step number, how many of the smallest trapezoids are there when  $n$  is 4? What about when  $n$  is 7?
  
  
  
  
  
  
  
  
  
  
2. At a certain step,  $k$ , there are 262,144 smallest trapezoids.
  - a. Write an equation to represent the relationship between  $k$  and the number of trapezoids in step  $k$ .
  
  
  
  
  
  
  
  
  
  
  - b. Explain to a partner how you might find the value of  $k$ .

## 8.3

## Successive Splitting



In a lab, a colony of 100 thousand bacteria is placed on a petri dish. The population grows exponentially, tripling every hour.

1. How would you estimate or find the population of bacteria in:
  - a. 4 hours?
  - b. 90 minutes?
  - c.  $\frac{1}{2}$  hour?
2. How would you estimate or find the number of hours it would take the population to grow to:
  - a. 1,000 thousand bacteria?
  - b. double the initial population?

 **Are you ready for more?**

A \$1,000 investment increases in value by 5% each year. About how many years does it take for the value of the investment to double? Explain how you know.

## 8.4 Missing Values

Complete the tables.

|       |                |               |               |   |               |   |   |    |   |     |       |
|-------|----------------|---------------|---------------|---|---------------|---|---|----|---|-----|-------|
| $x$   |                |               | -1            | 0 | $\frac{1}{2}$ | 1 |   |    | 5 |     |       |
| $2^x$ | $\frac{1}{32}$ | $\frac{1}{4}$ | $\frac{1}{2}$ |   |               |   | 4 | 16 |   | 256 | 1,024 |

|       |                |               |   |               |               |   |     |     |       |
|-------|----------------|---------------|---|---------------|---------------|---|-----|-----|-------|
| $x$   |                |               |   | $\frac{1}{3}$ | $\frac{1}{2}$ |   |     |     |       |
| $5^x$ | $\frac{1}{25}$ | $\frac{1}{5}$ | 1 |               |               | 5 | 125 | 625 | 3,125 |

Be prepared to explain how you found the missing values.

## Lesson 8 Summary

Sometimes we know the value of an exponential expression but we don't know the exponent that produces that value.

For example, suppose the population of a town was 1 thousand. Since then, the population has doubled every decade and is currently at 32 thousand. How many decades has it been since the population was 1 thousand?

If we say that  $d$  is the number of decades since the population was 1 thousand, then  $1 \cdot 2^d$ , or just  $2^d$ , represents the population, in thousands, after  $d$  decades. To answer the question, we need to find the exponent in  $2^d = 32$ . We can reason that since  $2^5 = 32$ , it has been 5 decades since the population was 1 thousand people.

When did the town have 250 people? Assuming that the doubling started before the population was measured to be 1 thousand, we can write:  $2^d = 0.25$ , or  $2^d = \frac{1}{4}$ . We know that  $2^{-2} = \frac{1}{4}$ , so the exponent  $d$  has a value of -2. The population was 250 two decades before it was 1,000.

But it may not always be so straightforward to calculate. For example, it is harder to tell the value of  $d$  in  $2^d = 805$  or in  $2^d = 4.5$ . In upcoming lessons, we'll learn more ways to find unknown exponents.