

# End Behavior (Part 1)

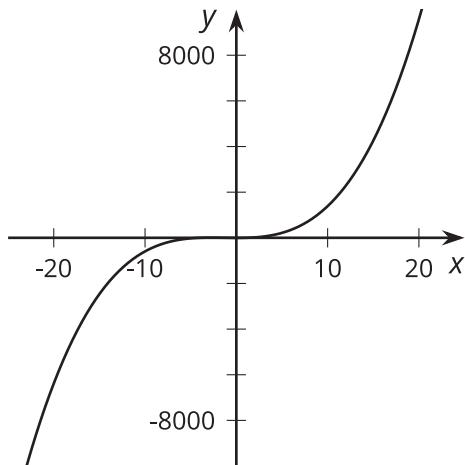
Let's investigate the shape of polynomials.

8.1

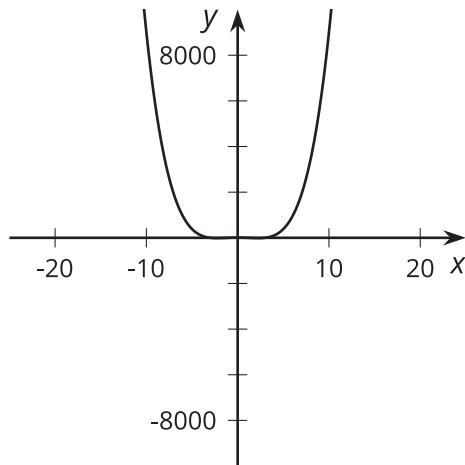
## Notice and Wonder: A Different View

What do you notice? What do you wonder?

$$y = x^3 + 4x^2 - x - 4$$



$$y = x^4 - 10x^2 + 9$$



## 8.2 Polynomial End Behavior

1. In the column for your assigned polynomial, evaluate for the different values of  $x$ . Discuss what you notice with your group.

$x$	$y = x^2 + 1$	$y = x^3 + 1$	$y = x^4 + 1$	$y = x^5 + 1$
-1000				
-100				
-10				
-1				
1				
10				
100				
1000				

2. Sketch what you think the **end behavior** of your polynomial looks like, then check your work using graphing technology.

## 💡 Are you ready for more?

Mai is studying the function  $p(x) = -\frac{1}{100}x^3 + 25,422x^2 + 8x + 26$ . She makes a table of values for  $p$  with  $x = \pm 1, \pm 5, \pm 10, \pm 20$  and thinks that this function has large positive output values in both directions on the  $x$ -axis. Do you agree with Mai? Explain your reasoning.

### 8.3

## Two Polynomial Equations

Consider the polynomial  $y = 2x^5 - 5x^4 - 30x^3 + 5x^2 + 88x + 60$ .

1. Identify the degree of the polynomial.
2. Which of the 6 terms,  $2x^5$ ,  $5x^4$ ,  $30x^3$ ,  $5x^2$ ,  $88x$ , or  $60$ , is greatest when:
  - a.  $x = 0$
  - b.  $x = 1$
  - c.  $x = 3$
  - d.  $x = 5$
3. Describe the end behavior of the polynomial.

## 👤 Lesson 8 Summary

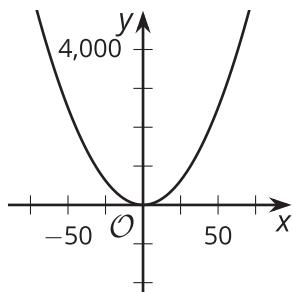
The value of the leading term determines the **end behavior** of the function, that is, how the outputs of the function change as we look at input values farther and farther from 0.

Consider the polynomial  $P(x) = x^4 - 30x^3 - 20x^2 + 1000$ . The leading term,  $x^4$ , almost seems smaller than the other three terms, and for certain values of  $x$ , this is even true. But, for values of  $x$  far away from 0, the leading term will always have the greatest value. In the case of  $P$ , as  $x$  gets larger and larger in the positive and negative directions, the output of the function gets larger and larger in the positive direction.

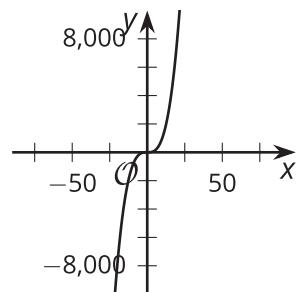
$x$	$x^4$	$-30x^3$	$-20x^2$	1000	$P(x)$
-500	62,500,000,000	3,750,000,000	-5,000,000	1,000	66,245,001,000
-100	100,000,000	30,000,000	-200,000	1,000	129,801,000
-10	10,000	30,000	-2,000	1,000	39,000
0	0	0	0	1,000	1000
10	10,000	-30,000	-2,000	1,000	-21,000
100	100,000,000	-30,000,000	-200,000	1,000	69,801,000
500	62,500,000,000	-3,750,000,000	-5,000,000	1,000	58,745,001,000

If we graph  $y = x^2$ ,  $y = x^3$  and  $y = x^4$  and zoom out, we see the following:

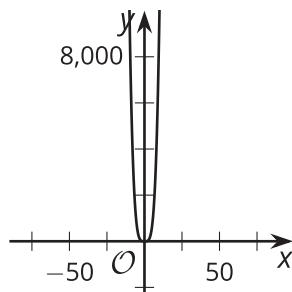
$$y = x^2$$



$$y = x^3$$



$$y = x^4$$



For both  $y = x^2$  and  $y = x^4$ , large positive values of  $x$  or large negative values of  $x$  each result in large positive values of  $y$ .

But for  $y = x^3$ , large positive values of  $x$  result in large positive values of  $y$ , while large negative values of  $x$  result in large negative values of  $y$ .