

# Finding Intersections

Let's think about two polynomials at once.

## 11.1 Math Talk: When $f$ Meets $g$

Find a point mentally where the graphs of the two functions intersect, if one exists.

- $f(x) = x$  and  $g(x) = 3$
- $j(x) = (x + 3)(x - 3)$  and  $k(x) = 0$
- $m(x) = (x + 3)(x - 3)$  and  $n(x) = (x - 3)$
- $p(x) = (x + 5)(x - 5)$  and  $q(x) = (x + 3)(x - 3)$

## 11.2

## More Points of Intersection

For each pair of polynomials given, find all points of intersection of their graphs.

1.  $c(x) = x^2 - 7$  and  $d(x) = 2$

2.  $f(x) = (x + 7)(x - 4)$  and  $g(x) = x - 4$

3.  $m(x) = (x + 7)(x - 4)$  and  $n(x) = (2x + 5)(x - 4)$

4.  $p(x) = (x + 1)(x - 8)$  and  $q(x) = (x + 2)(x - 4)$



## Are you ready for more?

Find all points of intersection of the graphs of the equations  $p(x) = (2x + 3)(x - 5)$  and  $q(x) = (x + 5)(x + 1)(x - 3)$ . Use graphing technology to check your solutions.

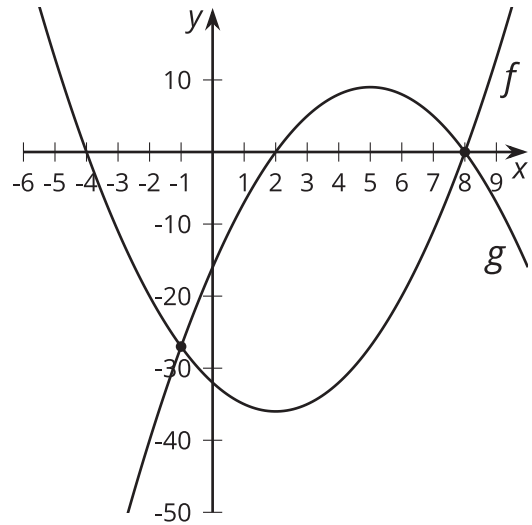
### 11.3 Graphing to Find Points of Intersection

Consider the functions  $p(x) = 5x^3 + 6x^2 + 4x$  and  $q(x) = 5640$ .

1. Use graphing technology to find a value of  $x$  that makes  $p(x) = q(x)$  true.
2. Using the  $x$ -value at the point of intersection, what is the value of  $5x^3 + 6x^2 + 4x - 5640$ ?
3. What does your answer suggest is a possible factor of  $5x^3 + 6x^2 + 4x - 5640$ ?
4. a. Write your own polynomial  $m(x)$  of degree 3 or higher.  
  
b. Use graphing technology to estimate the values of  $x$  that make  $m(x) = q(x)$  true.

## Lesson 11 Summary

When asked to find all values of  $x$  that make an equation like  $(x + 4)(x - 8) = (2 - x)(x - 8)$  true, one way to consider the question is to ask where the graphs of the functions  $f(x) = (x + 4)(x - 8)$  and  $g(x) = (2 - x)(x - 8)$  intersect.



Since the coordinate of any point of intersection has the form  $(a, f(a)) = (a, g(a))$ , these points must make  $f(x) = g(x)$  true when  $x = a$ . In our example, we can tell from the graph that both  $x = -1$  and  $x = 8$  are solutions to the original equation.

We can also use algebra to identify solutions to  $(x + 4)(x - 8) = (2 - x)(x - 8)$  by rearranging and then recognizing that both parts have a factor of  $(x - 8)$  in common:

$$\begin{aligned}(x + 4)(x - 8) &= (2 - x)(x - 8) \\(x + 4)(x - 8) - (2 - x)(x - 8) &= 0 \\(x - 8)(x + 4 - 2 + x) &= 0 \\(x - 8)(2x + 2) &= 0 \\x &= 8, -1\end{aligned}$$

For polynomials created to model specific situations that have a more complicated structure, solving without using technology can be challenging, especially because the graphs of two polynomials can intersect at multiple points.