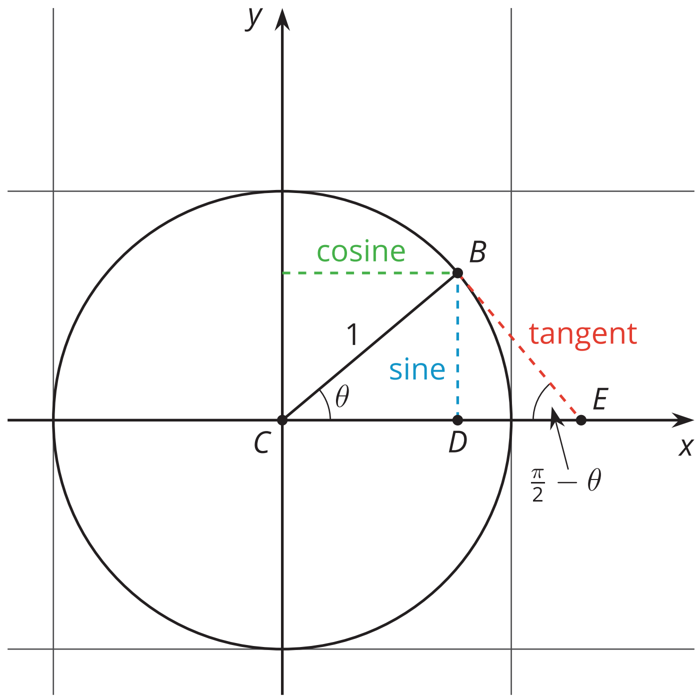
Unit 6, Lesson 13

# Some New Ratios

Let's explore graphs of some more trigonometric functions.

## 13.1 Notice and Wonder: Angles and Lines

What do you notice? What do you wonder?



# 13.2 Some Additional Lines

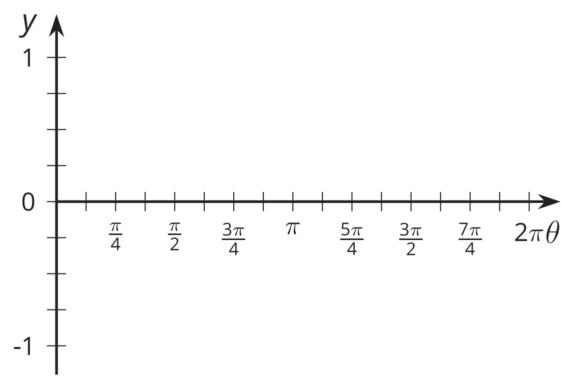
Your teacher will assign you a function—either secant, or cotangent, where $f(\theta) = \sec(\theta)$, $g(\theta) = \csc(\theta)$, and $h(\theta) = \cot(\theta)$.

1. Complete the table of values for your function from 0 to $2\pi$.

| $\theta$ | |
|---|---|
| 0 | |
| $\frac{\pi}{6}$ | |
| $\frac{\pi}{4}$ | |
| $\frac{\pi}{3}$ | |
| $\frac{\pi}{2}$ | |
| $\frac{2\pi}{3}$ | |
| $\frac{3\pi}{4}$ | |
| $\frac{5\pi}{6}$ | |
| $\pi$ | |

| $\theta$ | |
|---|---|
| $\frac{7\pi}{6}$ | |
| $\frac{5\pi}{4}$ | |
| $\frac{4\pi}{3}$ | |
| $\frac{3\pi}{2}$ | |
| $\frac{5\pi}{3}$ | |
| $\frac{7\pi}{4}$ | |
| $\frac{11\pi}{6}$ | |
| $2\pi$ | |

2. Graph your function from 0 to $2\pi$.




3. Create a display that includes your function name, the graph, the trigonometric ratio, and the reciprocal identity.



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## Are you ready for more?

The "co" in "cosine," "cotangent," and "cosecant" refers to the complement of the angle. The complement of $\theta$ is $\frac{\pi}{2} - \theta$.

Explain for each of the function pairs how you know that the statements are true:

1. $\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right)$
2. $\cot(\theta) = \tan\left(\frac{\pi}{2} - \theta\right)$
3. $\csc(\theta) = \sec\left(\frac{\pi}{2} - \theta\right)$

## 13.3 Card Sort: Periodic Functions

Your teacher will give you a set of cards. Take turns with your partner to match a graph with a function name, ratio, and identity.

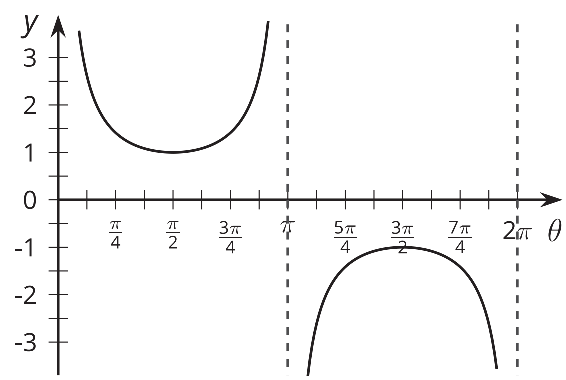
1. For each match that you find, explain to your partner how you know it's a match.
2. For each match that your partner finds, listen carefully to the explanation. If you disagree, discuss your thinking, and work to reach an agreement.

## Lesson 13 Summary

We can use the sine, cosine, and tangent functions to talk about some new trigonometric functions: cosecant, secant, and cotangent.

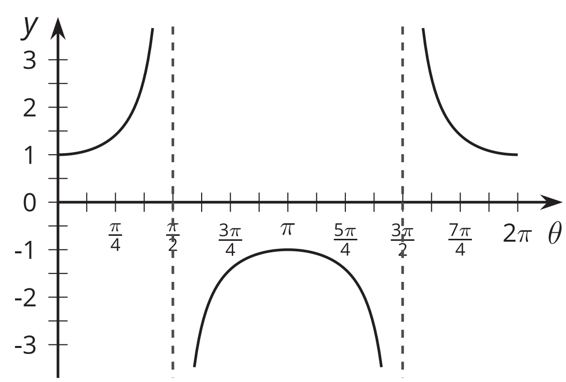
*Cosecant* is the reciprocal function of sine. This means that $\csc(\theta) = \frac{1}{\sin(\theta)}$. On a right triangle, we can identify the trigonometric ratio $\csc(\theta) = \frac{\text{hypotenuse}}{\text{opposite}}$. We can use this information to create a graph of $y = \csc(\theta)$. The graph will have an asymptote any time $\sin(\theta) = 0$. We also know that $y = 1$ any time $\sin(\theta) = 1$, $y = \text{-}1$ any time $\sin(\theta) = \text{-}1$, and has the same period as $\sin(\theta)$. Here is the graph of $y = \csc(\theta)$:

$y = \csc(\theta)$




*Secant* is the reciprocal function of cosine. This means that $\sec(\theta) = \frac{1}{\cos(\theta)}$. On a right triangle, we can identify the trigonometric ratio $\sec(\theta) = \frac{\text{hypotenuse}}{\text{adjacent}}$. We can use this information to create a graph of $y = \sec(\theta)$. The graph will have an asymptote any time $\cos(\theta) = 0$. We also know that $y = 1$ any time $\cos(\theta) = 1$, $y = \text{-}1$ any time $\cos(\theta) = \text{-}1$, and has the same period as $\cos(\theta)$. Here is the graph of $y = \sec(\theta)$:

$y = \sec(\theta)$



*Cotangent* is the reciprocal function of tangent. This means that $\cot(\theta) = \frac{1}{\tan(\theta)}$. On a right triangle, we can identify the trigonometric ratio $\cot(\theta) = \frac{\text{adjacent}}{\text{opposite}}$. We can use this information to create a graph of $y = \cot(\theta)$. The the graph will have an asymptote any time $\tan(\theta) = 0$, and it will have a value of 0 any time $\tan(\theta)$ has an asymptote. We also know that $y = 1$ any time $\tan(\theta) = 1$, $y = \text{-}1$ any time $\tan(\theta) = \text{-}1$, and has the same period as $\tan(\theta)$. Here is the graph of $y = \cot(\theta)$:

$y = \cot(\theta)$