



Solving Systems by Elimination (Part 2)

Let's think about why adding and subtracting equations work for solving systems of linear equations.

15.1 Is It Still True?

Here is a true equation: $50 + 1 = 51$.

1. Perform each of the following operations and answer these questions: What does each resulting equation look like? Is it still a true equation?
 - a. Add 12 to each side of the equation.
 - b. Add $10 + 2$ to the left side of the equation and 12 to the right side.
 - c. Add the equation $4 + 3 = 7$ to the equation $50 + 1 = 51$.
2. Write a new equation that, when added to $50 + 1 = 51$, gives a sum that is also a true equation.
3. Write a new equation that, when added to $50 + 1 = 51$, gives a sum that is a false equation.

15.2 Classroom Supplies

A teacher purchased 20 calculators and 10 measuring tapes for her class and paid \$495. Later, she realized that she didn't order enough supplies. She placed another order of 8 of the same calculators and 1 more of the same measuring tape and paid \$178.50.

This system represents the constraints in this situation:

$$\begin{cases} 20c + 10m = 495 \\ 8c + m = 178.50 \end{cases}$$



1. Discuss with a partner:
 - a. In this situation, what do the solutions to the first equation mean?
 - b. What do the solutions to the second equation mean?
 - c. For each equation, how many possible solutions are there? Explain how you know.
 - d. In this situation, what does the solution to the system mean?
 2. Find the solution to the system. Explain or show your reasoning.
-
3. To be reimbursed for the cost of the supplies, the teacher recorded: "Items purchased: 28 calculators and 11 measuring tapes. Amount: \$673.50."
 - a. Write an equation to represent the relationship between the numbers of calculators and measuring tapes, the prices of those supplies, and the total amount spent.
 - b. How is this equation related to the first two equations?

- c. In this situation, what do the solutions of this equation mean?
- d. How many possible solutions does this equation have? How many solutions make sense in this situation? Explain your reasoning.

15.3 A Bunch of Systems

Solve each system of equations without graphing and show your reasoning. Then, check your solutions.

$$A \begin{cases} 2x + 3y = 7 \\ -2x + 4y = 14 \end{cases}$$

$$B \begin{cases} 2x + 3y = 7 \\ 3x - 3y = 3 \end{cases}$$

$$C \begin{cases} 2x + 3y = 5 \\ 2x + 4y = 9 \end{cases}$$

$$D \begin{cases} 2x + 3y = 16 \\ 6x - 5y = 20 \end{cases}$$

 Are you ready for more?

This system has three equations:
$$\begin{cases} 3x + 2y - z = 7 \\ -3x + y + 2z = -14 \\ 3x + y - z = 10 \end{cases}$$

1. Add the first two equations to get a new equation.
2. Add the second two equations to get a new equation.
3. Solve the system of your two new equations.
4. What is the solution to the original system of equations?

Lesson 15 Summary

When solving a system with two equations, why is it acceptable to add the two equations, or to subtract one equation from the other?

Remember that an equation is a statement that says two things are equal. For example, the equation $a = b$ says a number a has the same value as another number b . The equation $10 + 2 = 12$ says that $10 + 2$ has the same value as 12.

If $a = b$ and $10 + 2 = 12$ are true statements, then adding $10 + 2$ to a and adding 12 to b means adding the same amount to each side of $a = b$. The result, $a + 10 + 2 = b + 12$, is also a true statement.

As long as we add an equal amount to each side of a true equation, the two sides of the resulting equation will remain equal.

We can reason the same way about adding variable equations in a system like this:

$$\begin{cases} d + f = 17 \\ -2d + f = -1 \end{cases}$$

In each equation, if (d, f) is a solution, the expression on the left of the equal sign and the number on the right are equal. Because $-2d + f$ is equal to -1 :

- Adding $-2d + f$ to $d + f$ and adding -1 to 17 means adding an equal amount to each side of $d + f = 17$. The two sides of the new equation, $-d + 2f = 16$, stay equal.

$$\begin{array}{r} d + f = 17 \\ -2d + f = -1 \quad + \\ \hline -d + 2f = 16 \end{array}$$

The d - and f -values that make the original equations true also make this equation true.

- Subtracting $-2d + f$ from $d + f$ and subtracting -1 from 17 means subtracting an equal amount from each side of $d + f = 17$. The two sides of the new equation, $3d = 18$, stay equal.

$$\begin{array}{r} d + f = 17 \\ -2d + f = -1 \quad - \\ \hline 3d = 18 \end{array}$$

The f -variable is eliminated, but the d -value that makes both the original equations true also makes this equation true.

From $3d = 18$, we know that $d = 6$. Because 6 is also the d -value that makes the original equations true, we can substitute it into one of the equations and find the f -value.

The solution to the system is $d = 6$, $f = 11$, or the point $(6, 11)$ on the graphs representing the system. If we substitute 6 and 11 for d and f in any of the equations, we will find true equations. (Try it!)

