The Paths of the Planets

Task Statement 1

In the early 1600s, Johannes Kepler (1571–1630) studied the motions of the planets to find a good mathematical model for them. In 1619, he published his third law of planetary motion, which describes how the orbital period of a planet is related to its distance from the Sun. In Kepler's time, Uranus and Neptune had not yet been discovered, but here is the data for all 8 planets:

planet	distance (millions of km)	period (days)
Mercury	57.9	88.0
Venus	108.2	224.7
Earth	149.6	365.2
Mars	227.9	687.0
Jupiter	778.6	4,331
Saturn	1,433.5	10,747
Uranus	2,872.5	30,589
Neptune	4,495.1	59,800

1. Plot the distance (x) and the period (y) of each planet, and find a polynomial model that fits the data as well as possible. You may have to experiment with both the degree of the polynomial function and the number of terms.
2. Make another plot that uses the square root of each distance instead, and find a polynomial model that fits those points as well as possible.
3. Which model do you think is best?
4. a. Jupiter has a lot of moons. Look up the periods and the distances of some of them, and use that data to make a polynomial model of the relationship between the period and the distance for Jupiter's moons.

   b. Use your model to predict the period of another of Jupiter's moons, using the radius of its orbit.

   c. How good was the prediction? What are some possible sources of error?

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Task Statement 2

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1. Plot the distance (x) and the period (y) of each planet, and find a polynomial model that fits the data as well as possible. You may have to experiment with both the degree of the polynomial function and the number of terms.
2. Make another plot that uses the square root of each distance instead, and find a polynomial model that fits those points as well as possible.
3. Which model do you think is best?

4. a. Jupiter has a lot of moons. Here are the periods and the distances of the Galilean moons, which were discovered in 1610:

moon	distance (thousands of km)	period (days)
Io	421.8	1.77
Europa	671.1	3.55
Ganymede	1,070.4	7.16
Callisto	1,882.7	16.69

Use the data to make a polynomial model of the relationship between the period and the distance for Jupiter's moons.

b. Another moon of Jupiter, number LXXI, was discovered in 2018. Its distance from Jupiter is 11,483 thousand km. According to your model, what should be its period?

c. The actual period of the moon is 252.0 days. How close was the prediction? What are some possible sources of error?