What about Other Bases?

Let's explore exponent patterns with bases other than 10.

6.1 Math Talk: Comparing Expressions with Exponents

Decide mentally whether each statement is true.

- $3^5 < 4^6$
- $(-3)^2 < 3^2$
- $(-3)^3 = 3^3$
- $(-5)^2 > -5^2$

6.2 What Happens with Zero and Negative Exponents?

Complete the table.

$\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$ $\cdot 2$

value	16					$\frac{1}{2}$			
exponent form	2^4								

$\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$ $\cdot ?$

1. As you move toward the left, each number is being multiplied by 2. What is the multiplier as you move toward the right?

2. Use the patterns you found in the table to write 2^{-6} as a fraction.

3. Write $\frac{1}{32}$ as a power of 2 with a single exponent.

4. What is the value of 2^0?

5. From the work you have done with negative exponents, how would you write 5^{-3} as a fraction?

6. How would you write 3^{-4} as a fraction?

Are you ready for more?

1. Find an expression equivalent to $\left(\frac{2}{3}\right)^{-3}$ but with positive exponents.

2. Find an expression equivalent to $\left(\frac{4}{5}\right)^{-8}$ but with positive exponents.

3. What patterns do you notice when you start with a fraction raised to a negative exponent and rewrite it using a single positive exponent? Show or explain your reasoning.

6.3 Exponent Rules with Bases Other than 10

Lin, Noah, Diego, and Elena each chose an expression to start with and then came up with a new list of expressions — some of which are equivalent to the original and some of which are not.

Choose 2 of the 4 lists to analyze. For each list of expressions you choose to analyze, decide which expressions are *not* equivalent to the original. Be prepared to explain your reasoning.

1. Lin's original expression is 5^{-9} and her list is:

   $(5^3)^{-3}$ $\quad -5^9$ $\quad \frac{5^{-6}}{5^3}$ $\quad (5^3)^{-2}$ $\quad \frac{5^{-4}}{5^{-5}}$ $\quad 5^{-4} \cdot 5^{-5}$

2. Noah's original expression is 3^{10} and his list is:

   $3^5 \cdot 3^2$ $\quad (3^5)^2$ $\quad (3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)$

   $\left(\frac{1}{3}\right)^{-10}$ $\quad 3^7 \cdot 3^3$ $\quad \frac{3^{20}}{3^{10}}$ $\quad \frac{3^{20}}{3^2}$

3. Diego's original expression is x^4 and his list is:

   $\frac{x^8}{x^4}$ $\quad x \cdot x \cdot x \cdot x$ $\quad \frac{x^{-4}}{x^{-8}}$ $\quad \frac{x^{-4}}{x^8}$

   $(x^2)^2$ $\quad 4 \cdot x$ $\quad x \cdot x^3$

4. Elena's original expression is 8^0 and her list is:

   1 $\quad 0$ $\quad 8^3 \cdot 8^{-3}$

   $\frac{8^2}{8^2}$ $\quad 10^0$ $\quad 11^0$

Lesson 6 Summary

We can keep track of repeated factors using exponent rules. These rules also help us make sense of negative exponents and why a number to the power of 0 is defined as 1. These rules can be written symbolically where the base a can be any positive number:

Rule	Example showing how it works
$a^n \cdot a^m = a^{n+m}$	$a^2 \cdot a^3 = (a \cdot a) \cdot (a \cdot a \cdot a) = a^5$ two factors that are a $\cdot$ three factors that are a = five factors that are a
$\left(a^n\right)^m = a^{n \cdot m}$	$\left(a^2\right)^3 = \underline{(a \cdot a)} \cdot \underline{(a \cdot a)} \cdot \underline{(a \cdot a)} = a^6$ three groups of two factors that are a = six factors that are a
$\frac{a^n}{a^m} = a^{n-m}$	$\frac{a^5}{a^2} = \frac{a \cdot a \cdot a \cdot a \cdot a}{a \cdot a} = \frac{a \cdot a}{a \cdot a} \cdot a \cdot a \cdot a = 1 \cdot a^3 = a^3$ five factors that are a $\div$ two factors that are a = three factors that are a
$a^0 = 1$	$\frac{a^6}{a^0} = a^{6-0} = a^6$ this value must be equal to 1
$a^{-n} = \frac{1}{a^n}$	$a^{-3} = \frac{1}{a} \cdot \frac{1}{a} \cdot \frac{1}{a} = \frac{1}{a^3}$ three factors that are one over a