Unit 4 Family Support Materials

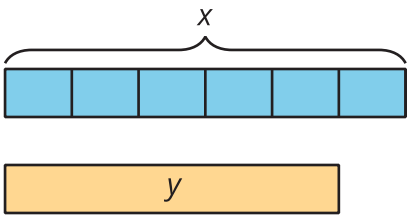
Proportional Relationships and Percentages

Section A: Proportional Relationships with Fractions

This week your student is learning about proportional relationships that involve fractions and decimals. For example, a baker decides to start using $\frac{1}{6}$ less than the amount of sugar called for in each recipe. If the recipe calls for 2 cups of sugar, the baker will leave out $\frac{1}{6} \cdot 2$, or $\frac{1}{3}$, cup of sugar. That means the baker will use only $2 - \frac{1}{3}$, or $1\frac{2}{3}$, cups of sugar.

amount of sugar in the recipe (x)	amount of sugar the baker uses (y)
1 cup	$\frac{5}{6}$ cup
$1\frac{1}{2}$ cups	$1\frac{1}{4}$ cups
2 cups	$1\frac{2}{3}$ cups

The amount of sugar the baker actually uses, y, is proportional to the amount of sugar called for in the recipe, x. The constant of proportionality is $\frac{5}{6}$.



$$y = x - \frac{1}{6}x$$

$$y = (1 - \frac{1}{6})x$$

$$y = \frac{5}{6}x$$

Another way to write this equation is $y = 0.8\overline{3}x$. The line above the 3 tells us that if we use **long division** to divide $5 \div 6$, we will keep getting the answer 3 over and over. This is an example of a **repeating decimal**.

Here is a task to try with your student:

The baker also decides to start using $\frac{1}{6}$ more than the amount of liquid called for in each recipe.

1. How much of each ingredient will the baker use if the recipe calls for:
   a. $1\frac{1}{2}$ cups of milk?

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b. 3 tablespoons of oil?

2. What is the constant of proportionality for the relationship between the amount of liquid called for in the recipe and the amount this baker uses?

Solution:

1. a. $1\frac{3}{4}$ cups, because $\frac{1}{6} \cdot 1\frac{1}{2} = \frac{1}{4}$ and $1\frac{1}{2} + \frac{1}{4} = 1\frac{3}{4}$

   b. $3\frac{1}{2}$ tablespoons, because $(1 + \frac{1}{6}) \cdot 3 = 3\frac{1}{2}$

2. $\frac{7}{6}$ or $1.1\overline{6}$. (Equivalent forms of these numbers are also acceptable solutions.)

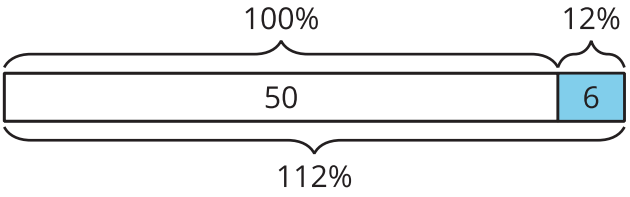
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Section B: Percent Increase and Decrease

This week, your student is learning to describe increases and decreases as a percentage of the starting amount. For example, two different school clubs can gain the same number of students but have different **percent increases**.

The cooking club has 50 students. Then they gain 6 students.

This is a 12% increase, because $6 \div 50 = 0.12$.

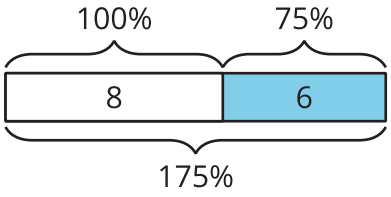


They now have 56 students, which is 112% of the starting amount.

$$1.12 \cdot 50 = 56$$

The computer club has 8 students. Then they gain 6 students.

This is a 75% increase, because $6 \div 8 = 0.75$.



They now have 14 students, which is 175% of the starting amount.

$$1.75 \cdot 8 = 14$$

Here is a task to try with your student:

The photography club has 20 students. Then the number of students increases by 35%. How many students are in the photography club now?

Solution:

27 students. Possible strategies:

- The club gains 7 new students, because $0.35 \cdot 20 = 7$. The club now has 27 students, because $20 + 7 = 27$.
- The club now has 135% as many students as they started with, because $100 + 35 = 135$. That means they have 27 students, because $1.35 \cdot 20 = 27$.

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Section C: Applying Percentages

This week, your student is learning about real-world situations that use percent increase and percent decrease, such as tax, interest, markup, and discounts.

For example, the price tag on a jacket says $24. The customer must also pay a sales tax equal to 7.5% of the price. What is the total cost of the jacket, including tax?

$$24 \cdot 1.075 = 25.80$$

The customer will pay 107.5% of the price listed on the tag, so the total cost will be $25.80.

We can also find the percentage. For example, a backpack originally costs $22.50, but it is on sale for $18.99. The discount is what percentage of the original price?

$$\begin{aligned} 22.50x &= 18.99 \\ x &= 18.99 \div 22.50 \\ x &= 0.844 \end{aligned}$$

The sale price is 84.4% of the original price. The discount is $(100 - 84.4)$%, or 15.6%, of the original price.

Here is a task to try with your student:

A restaurant bill is $18.75. If you pay $22, the tip that you leave for the server is what percentage of the bill?

Solution:

$17.\overline{3}\%$. Possible strategy: You pay $117.\overline{3}\%$ of the bill, because $22 \div 18.75 = 1.17\overline{3}$. You leave a $17.\overline{3}\%$ tip, because $117.\overline{3} - 100 = 17.\overline{3}$.

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