Edge Lengths, Volumes, and Cube Roots

Let's explore the relationship between volume and edge lengths of cubes.

12.1 Ordering Squares and Cubes

Let a, b, c, d, e, and f be positive numbers.

Given these equations, arrange a, b, c, d, e, and f from least to greatest. Explain your reasoning.

- $a^2 = 9$
- $b^3 = 8$
- $c^2 = 10$
- $d^3 = 9$
- $e^2 = 8$
- $f^3 = 7$

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12.2 Card Sort: Rooted in the Number Line

Your teacher will give you a set of cards. Each card has a number line with a plotted point, an equation, or a square or **cube root** value.

For each card with a letter and square or cube root value, match it with the location on a number line where the value exists, and the equation that the value makes true. Record your matches and be prepared to explain your reasoning.

12.3 Cube Root Values

The value of a cube root of a number lies between two integers. Which are those consecutive whole numbers for the following? Be prepared to explain your reasoning.

1. $\sqrt[3]{5}$
2. $\sqrt[3]{23}$
3. $\sqrt[3]{81}$
4. $\sqrt[3]{999}$

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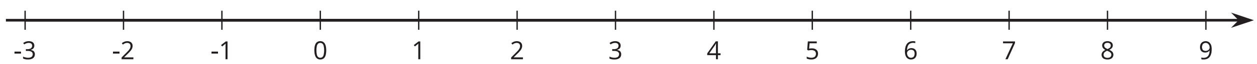
12.4 Solutions on a Number Line

The numbers x, y, and z are positive, and:

$x^3 = 5$ $\qquad$ $y^3 = 27$ $\qquad$ $z^3 = 700$



1. Plot x, y, and z on the number line. Be prepared to share your reasoning with the class.
2. Plot $-\sqrt[3]{2}$ on the number line.

Are you ready for more?

Diego knows that $8^2 = 64$ and that $4^3 = 64$. He says that this means the following are all true:

- $\sqrt{64} = 8$
- $\sqrt[3]{64} = 4$
- $\sqrt{-64} = -8$
- $\sqrt[3]{-64} = -4$

Is he correct? Explain how you know.

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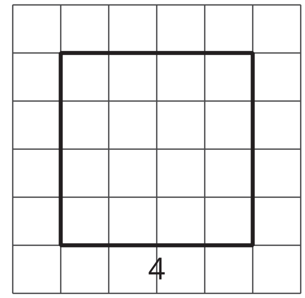
Lesson 12 Summary

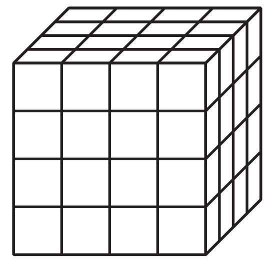
For a square, its side length is the square root of its area. For example, this square has an area of 16 square units and a side length of 4 units.



Both of these equations are true:

$$4^2 = 16 \qquad \sqrt{16} = 4$$

For a cube, the edge length is the **cube root** of its volume. For example, this cube has a volume of 64 cubic units and an edge length of 4 units:



Both of these equations are true:

$$4^3 = 64 \qquad \sqrt[3]{64} = 4$$

$\sqrt[3]{64}$ is pronounced "the cube root of 64."

Like square roots, most cube roots of whole numbers are irrational. The only time the cube root of a number is a rational number is when the number we are taking the cube root of is a perfect cube. For example, 8 is a perfect cube, and $\sqrt[3]{8} = 2$.

We can approximate the values of the cube root of a number by observing the integers around it and remembering the relationship between cubes and cube roots. For example, $\sqrt[3]{20}$ is between 2 and 3 since $2^3 = 8$ and $3^3 = 27$, and 20 is between 8 and 27. Similarly, since 100 is between 4^3 and 5^3, we know $\sqrt[3]{100}$ is between 4 and 5.

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