Unit 2, Lesson 8

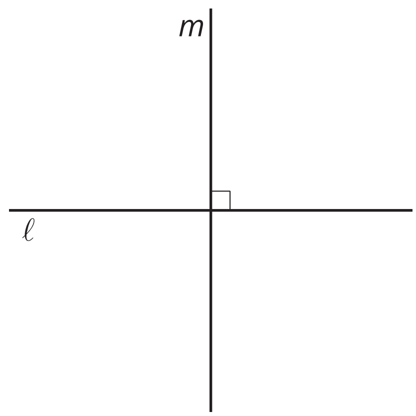
The Perpendicular Bisector Theorem

Let’s convince ourselves that what we’ve conjectured about perpendicular bisectors must be true.

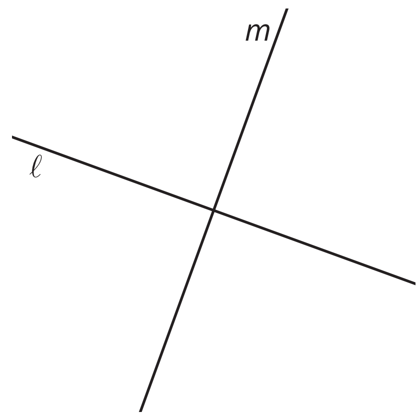
8.1 Which Three Go Together: Intersecting Lines

Which three go together? Why do they go together?

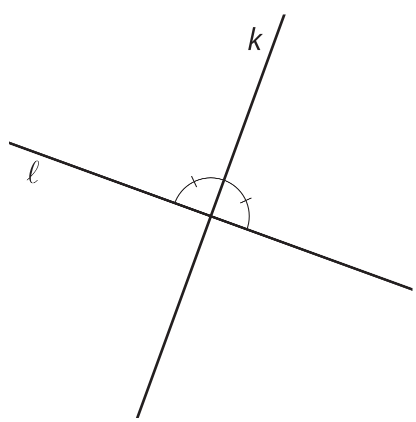
A



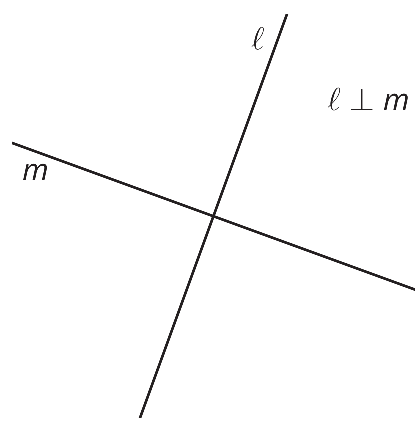
B



C



D

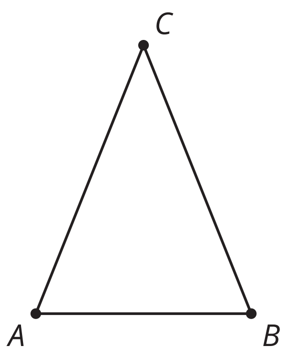


8.2 Lots of Lines (Part 1)

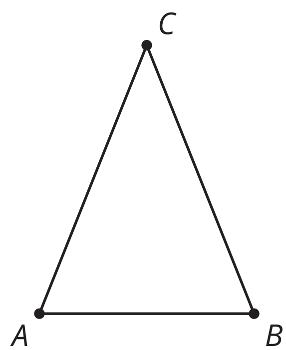
Diego, Jada, and Noah were given the following task: Prove that if a point C is the same distance from A as it is from B, then C must be on the perpendicular bisector of AB.

Read the script your teacher will give you. After each sentence you read, decide if there is anything to add to each diagram (and if so, add it).

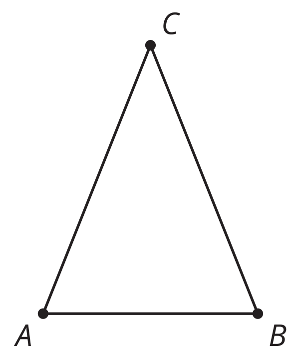
Diego’s image:



Jada’s image:



Noah’s image:

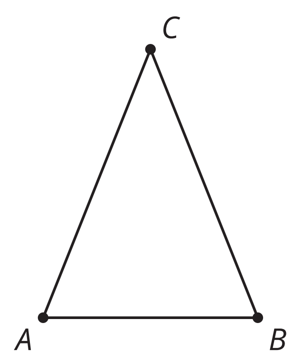


With your group, choose one student’s approach to discuss.

1. What do you notice that this student understands about the problem?
2. What question would you ask them to help them move forward?

8.3 Lots of Lines (Part 2)

Prove that if a point C is the same distance from A as it is from B, then C must be on the perpendicular bisector of AB.



Are you ready for more?

Elena has another approach: "I drew the line of reflection. If you reflect across C, then A and B will switch places, meaning A' coincides with B, and B' coincides with A. C will stay in its place, so the triangles will be congruent."

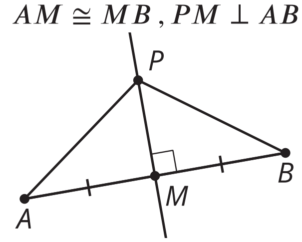
1. What feedback would you give Elena?

2. Write your own explanation based on Elena's idea.

8.4 Not Too Close, Not Too Far

1. Work on your own to write a rough draft of a proof for the statement: If P is a point on the perpendicular bisector of AB, prove that the distance from P to A is the same as the distance from P to B.

$\overline{AM} \cong \overline{MB}$, $\overline{PM} \perp \overline{AB}$



2. With your partner, discuss each other's drafts. Record your partner's feedback for your proof.
   - What do you notice that your partner understands about the problem?
   - What question would you ask them to help them move forward?

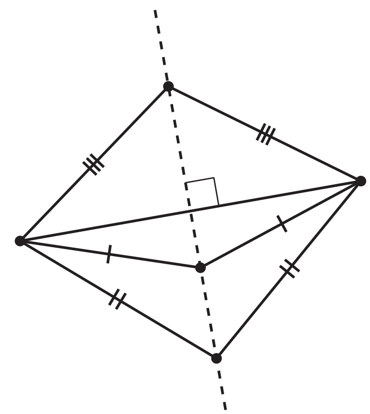
Lesson 8 Summary

The perpendicular bisector of a line segment is exactly those points that are the same distance from both endpoints of the line segment. This idea can be broken down into two statements:

- If a point is on the perpendicular bisector of a segment, then it must be the same distance from both endpoints of the line segment.
- If a point is the same distance from both endpoints of a line segment, then it must be on the perpendicular bisector of the segment.

These statements are converses of each other. Two statements are **converses** if the "if" parts and the "then" parts are swapped. The converse of a true statement isn't always true, but in this case, both statements of the Perpendicular Bisector Theorem are true.

A line of reflection is the perpendicular bisector of segments connecting points in the original figure with corresponding points in the image. Therefore, the following three lines are all the same:



- The perpendicular bisector of a segment.
- The set of points equidistant from the 2 endpoints of a segment.
- The line of reflection that takes the 2 endpoints of the segment onto each other and the segment onto itself.

It is useful to know that the perpendicular bisector of a line segment is also all the points which are the same distance from both endpoints of the line segment, because then:

- If 2 points A and B are both equidistant from the endpoints of a segment, then the line through A and B must be the perpendicular bisector of the segment (because 2 points define a unique line).
- If 2 points A and B are both equidistant from the endpoints of a segment, then the line through A and B must be the line of reflection that takes the segment to itself and swaps the endpoints.
- If a point A is on the line of reflection, then the distance from A to a point on the original figure is the same as the distance from A to its corresponding point on the image.
- If a point A is on the perpendicular bisector of a segment, then it is the same distance from A to both endpoints of the segment.