Transforming from an Original Function

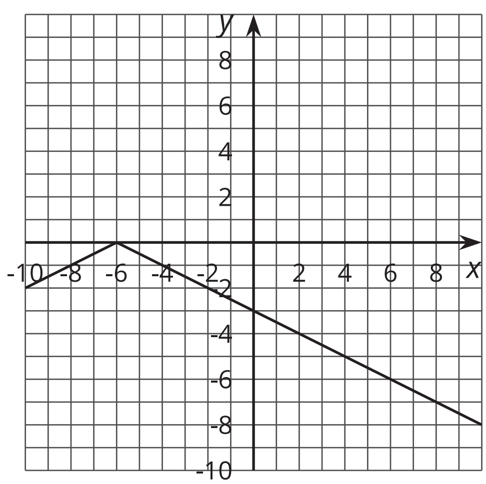
Let's transform some functions.

11.1 Which Three Go Together: Transformed Functions

Which three go together? Why do they go together?

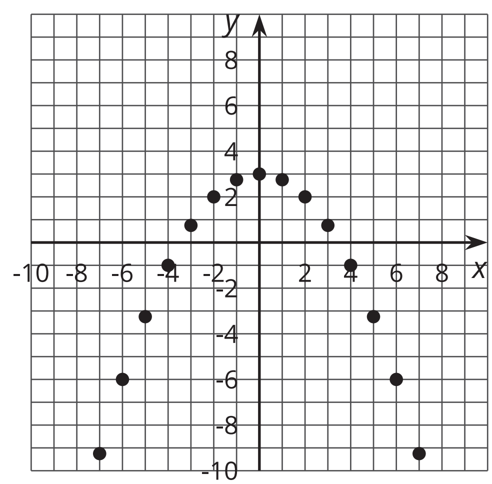
A

$y = -\left|\frac{1}{2}x + 3\right|$



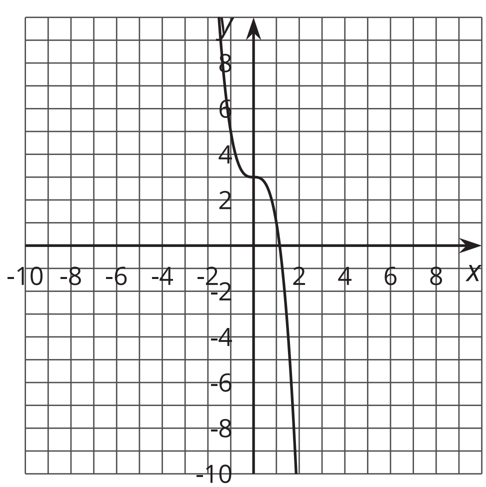
B

$y = -\left(\frac{1}{2}x\right)^2 + 3$, x is an integer



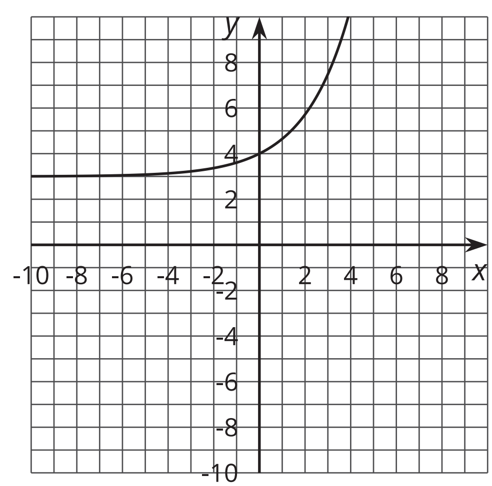
C

$y = -2x^3 + 3$



D

$y = e^{\left(\frac{1}{2}x\right)} + 3$



11.2 Comparing Transformed Functions

Your teacher will assign you an original function. Create a display that includes each of these steps:

1. Graph the original function in the coordinate plane. Plot and label the vertical intercept.
2. Transform the function by a translation down 3 and horizontal stretch by a factor of 2.
   a. Graph this transformed function in the same coordinate plane using a different color.
   b. Identify and label the image of the point from the original function onto the transformed function.
3. Write the equation of the transformed function.

11.3 Changing Graphs

1. Each person will choose a function from Column A and a transformation from Column B. Don't tell your partner what you chose!
2. Graph the transformed function on the grid paper.
3. When you and your partner have both drawn your functions, trade papers.
4. Identify the original function and transformation that your partner chose.
5. Write the equation of the transformed function on the graph, then take turns explaining to your partner how you know.

Column A

- $f(x) = x^2$
- $g(x) = 3^x$
- $h(x) = \sqrt[3]{x}$
- $k(x) = \frac{1}{x}$

Column B

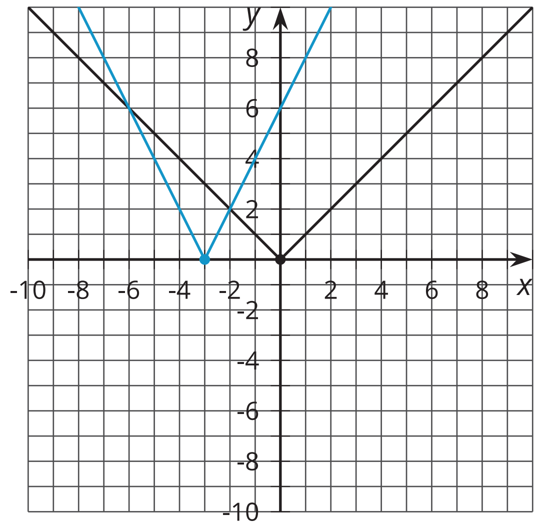
- Reflect over the x-axis and translate up 4.
- Horizontal stretch by a factor of 3 and reflect over the y-axis.
- Translate left 5 and a vertical compression by a factor of $\frac{1}{2}$.
- Horizontal stretch by a factor of 2 and vertical stretch by a factor of 2.

Lesson 11 Summary

When we apply transformations to functions, there are some patterns that we can see regardless of what type of function we started with. For example, let's apply this transformation to two different functions: shift left 3 and stretch vertically by a factor of 2.

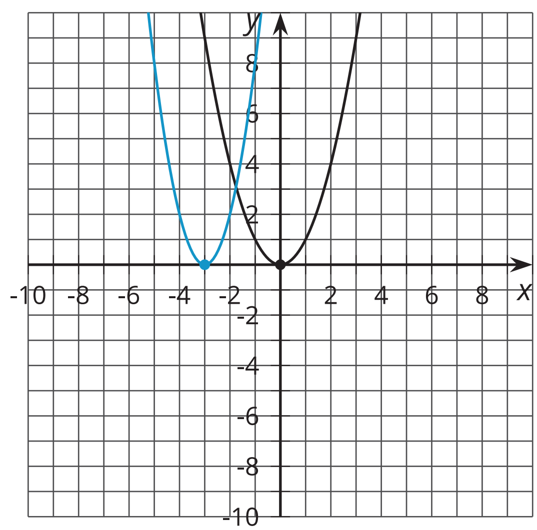
Let's start with an original function $f(x) = |x|$. Here are graphs of the original function and the transformed function:

We can see that the vertex has shifted left to the point $(-3, 0)$. We can also see that the slope of the piece that is decreasing is now -2, and the slope of the piece that is increasing is 2. This means we can write an equation for the transformed function: $y = 2|x + 3|$.



Now let's try a different original function, $g(x) = x^2$, but use the same transformation. Here are graphs of this original function and the transformed function:

We can see that the vertex of this function has also shifted left to the point $(-3, 0)$. The graph is narrower than the original because of the vertical stretch. An equation for this function is: $y = 2(x + 3)^2$.



The transformation changed each original function in the same ways: shifting left 3 and then stretching. This means we can make some observations about the changes in the equations. When the transformation affects the graph horizontally, such as a horizontal shift, reflection, or stretch, then it affects the input of the function. The changes to the equation will be grouped with the input as well. When the transformation affects the graph vertically, such as vertical shift, reflection, or stretch, then it affects the output of the function.

This makes sense, since the horizontal axis represents the input of the function, and the vertical axis represents the output of the function.