

Negative Rational Exponents

Let's investigate negative exponents.

5.1

Math Talk: Don't Be Negative

Find the value of each expression mentally.

- 9^2

- 9^{-2}

- $9^{\frac{1}{2}}$

- $9^{-\frac{1}{2}}$

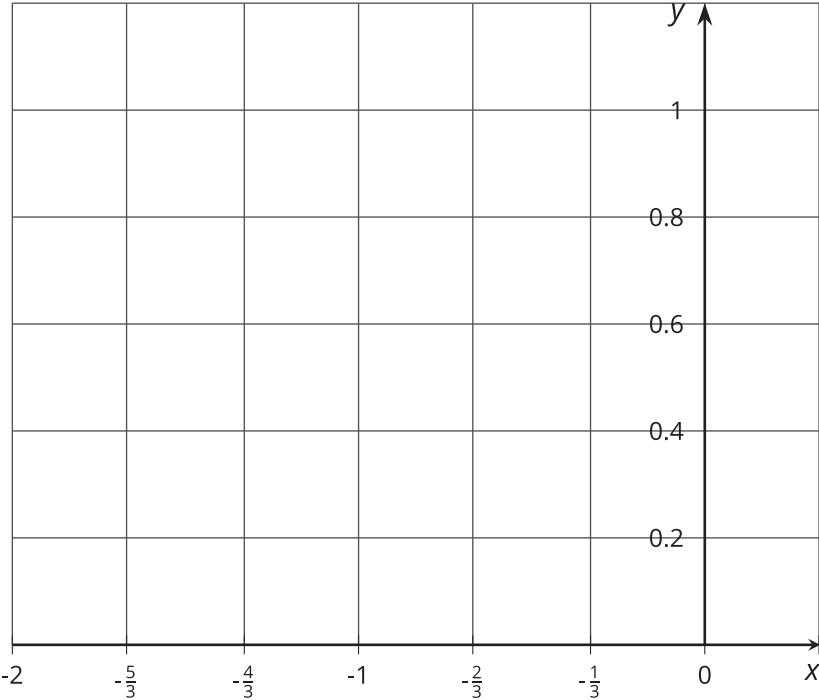
5.2

Negative Fractional Powers Are Just Numbers

1. Complete the table as much as you can without using a calculator. (You should be able to fill in three spaces.)

| | | | | | | | |
|-------------------------------|----------|--------------------|--------------------|----------|--------------------|--------------------|-------|
| x | -2 | $-\frac{5}{3}$ | $-\frac{4}{3}$ | -1 | $-\frac{2}{3}$ | $-\frac{1}{3}$ | 0 |
| 2^x (using exponents) | 2^{-2} | $2^{-\frac{5}{3}}$ | $2^{-\frac{4}{3}}$ | 2^{-1} | $2^{-\frac{2}{3}}$ | $2^{-\frac{1}{3}}$ | 2^0 |
| 2^x (decimal approximation) | | | | | | | |

- Plot these powers of 2 in the coordinate plane.
- Connect the points as smoothly as you can.
- Use your graph of $y = 2^x$ to estimate the value of the other powers in the table, and write your estimates in the table.



2. Let's investigate $2^{-\frac{1}{3}}$.

- Write $2^{-\frac{1}{3}}$ using radical notation.
- Use exponent rules to rewrite $\left(2^{-\frac{1}{3}}\right)^3$ in a simpler way.
- Raise your estimate of $2^{-\frac{1}{3}}$ to the third power. What should it be? How close did you get?

3. Let's investigate $2^{-\frac{2}{3}}$.

- Write $2^{-\frac{2}{3}}$ using radical notation.
- Use exponent rules to rewrite $\left(2^{-\frac{2}{3}}\right)^3$ in a simpler way.
- Raise your estimate of $2^{-\frac{2}{3}}$ to the third power. What should it be? How close did you get?

5.3

Any Fraction Can Be an Exponent

1. For each set of 3 numbers, cross out the expression that is not equal to the other two expressions.

- $8^{\frac{4}{5}}, \sqrt[4]{8^5}, \sqrt[5]{8^4}$
- $8^{-\frac{4}{5}}, \frac{1}{\sqrt[5]{8^4}}, -\frac{1}{\sqrt[5]{8^4}}$
- $\sqrt{4^3}, 4^{\frac{3}{2}}, 4^{\frac{2}{3}}$
- $\frac{1}{\sqrt{4^3}}, -4^{\frac{3}{2}}, 4^{-\frac{3}{2}}$

2. For each expression, write an equivalent expression with only positive, whole-number exponents.

- $17^{\frac{3}{2}}$
- $31^{-\frac{3}{2}}$

3. For each expression, write an equivalent expression in the form a^b .

- $(\sqrt{3})^4$
- $\frac{1}{(\sqrt[3]{5})^6}$

 **Are you ready for more?**

Write two different expressions that involve only roots and powers of 2 that are equivalent to $\frac{4^{\frac{3}{2}}}{8^{\frac{1}{4}}}$.

5.4**Make These Exponents Less Complicated**

Group expressions according to whether they are equal. Be prepared to explain your reasoning.

$$(\sqrt{3})^4$$

$$\sqrt{3^2}$$

$$\left(3^{\frac{1}{2}}\right)^4$$

$$(\sqrt{3})^2 \cdot (\sqrt{3})^2$$

$$(3^2)^{\frac{1}{2}}$$

$$3^2$$

$$3^{\frac{4}{2}}$$

$$\left(3^{\frac{1}{2}}\right)^2$$

Lesson 5 Summary

When we have a number with a negative exponent, it means we need to find the reciprocal of the number with the exponent that has the same magnitude, but is positive. Here are two examples:

$$7^{-5} = \frac{1}{7^5}$$

$$7^{-\frac{6}{5}} = \frac{1}{7^{\frac{6}{5}}}$$

The table shows a few more examples of exponents that are fractions and their radical equivalents.

| | | | | | | | |
|--------------------------------|---------------|---|-------------------------|-------|-------------------|-----------------------------------|-------|
| x | -1 | $-\frac{2}{3}$ | $-\frac{1}{3}$ | 0 | $\frac{1}{3}$ | $\frac{2}{3}$ | 1 |
| 5^x (using exponents) | 5^{-1} | $5^{-\frac{2}{3}}$ | $5^{-\frac{1}{3}}$ | 5^0 | $5^{\frac{1}{3}}$ | $5^{\frac{2}{3}}$ | 5^1 |
| 5^x (equivalent expressions) | $\frac{1}{5}$ | $\frac{1}{\sqrt[3]{5^2}}$ or $\frac{1}{\sqrt[3]{25}}$ | $\frac{1}{\sqrt[3]{5}}$ | 1 | $\sqrt[3]{5}$ | $\sqrt[3]{5^2}$ or $\sqrt[3]{25}$ | 5 |