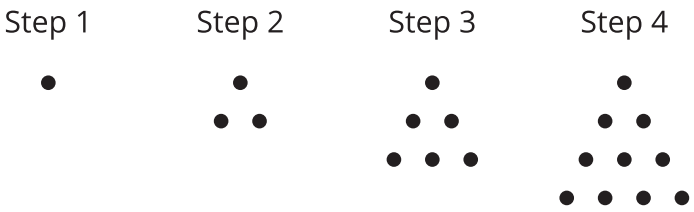
Unit 1, Lesson 5

Sequences Are Functions

Let's learn how to define a sequence recursively.

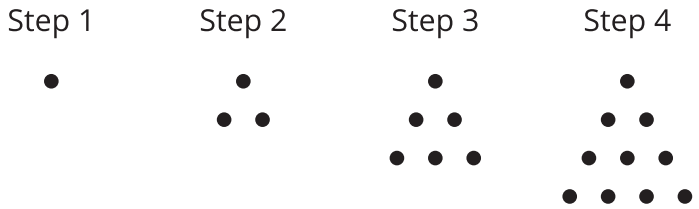
5.1 Notice and Wonder: Bowling for Triangles (Part 1)

What do you notice? What do you wonder?



5.2 Bowling for Triangles (Part 2)

Here is a visual pattern of dots. The number of dots $D(n)$ is a function of the step number n.



1. What values make sense for n in this situation? What values don't make sense for n?

2. Complete the table for Steps 1 to 5.

n	$D(n)$
1	1
2	$D(1)+2=3$
3	$D(2)+3=6$
4	
5	

3. Following the pattern in the table, write an equation for $D(n)$ in terms of the previous step. Be prepared to explain your reasoning.

Are you ready for more?

Consider the same triangular pattern.

1. Is the sequence defined by the number of dots in each step arithmetic, geometric, or neither? Explain how you know.

2. If possible, write an expression for the number of dots in Step n without using the value of D from a previous step.

5.3 Let's Define Some Sequences

Use the first 5 terms of each sequence to state if the sequence is arithmetic, geometric, or neither. Next, define the sequence recursively using function notation.

1. A: 30, 40, 50, 60, 70, . . .

2. B: 80, 40, 20, 10, 5, 2.5, . . .

3. C: 1, 2, 4, 8, 16, 32, . . .

4. E: $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

5. F: 20, 13, 6, -1, -8, . . .

6. G: 1, 3, 7, 15, 31, . . .

Lesson 5 Summary

Sometimes we can define a sequence recursively. That is, we can describe how to calculate the next term in a sequence if we know the previous term.

Here's a sequence: 6, 10, 14, 18, 22, . . . This is an arithmetic sequence, where each term is 4 more than the previous term. Since sequences are functions, let's call this sequence f, and then we can use function notation to write $f(n) = f(n-1) + 4$. Here, $f(n)$ is the term, $f(n-1)$ is the previous term, and + 4 represents the rate of change since f is an arithmetic sequence.

When we define a function recursively, we also must say what the first term is. Without that, there would be no way of knowing if the sequence defined by $f(n) = f(n-1) + 4$ started with 6 or 81 or some other number. Here, one possible starting term is $f(1) = 6$. It is possible to start sequences with input values other than 1, and the starting value often depends on what the sequence represents.

Combining this information gives the recursive definition for f: $f(1) = 6$ and $f(n) = f(n-1) + 4$ for $n \geq 2$, where n is an integer. We include the $n \geq 2$ at the end since the value of f at 1 is already given and the other terms in the sequence are generated by inputting integers larger than 1 into the definition.