

# Completing the Square (Part 3)

Let's complete the square for some more complicated expressions.

## 14.1

## Perfect Squares in Two Forms

Previously, we saw that  $(x + 3)^2$  can be expanded to standard form as  $x^2 + 2 \cdot 3x + 3^2$ .

1. Expand  $(5x + 3)^2$  into standard form.

2. Be prepared to share a conjecture about the relationship between the coefficients 5 and 3 in the factored form and the values in standard form.

## 14.2 Perfect in a Different Way

1. Write each expression in standard form:

a.  $(4x + 1)^2$

b.  $(5x - 2)^2$

c.  $(\frac{1}{2}x + 7)^2$

d.  $(3x + n)^2$

e.  $(kx + m)^2$

2. Decide if each expression is a perfect square. If so, write an equivalent expression of the form  $(kx + m)^2$ . If not, suggest one change to turn it into a perfect square.

a.  $4x^2 + 12x + 9$

b.  $4x^2 + 8x + 25$

## 14.3 When All the Stars Align

- Find the value of  $c$  to make each expression in the left column a perfect square in standard form. Then, write an equivalent expression in the form of squared factor. In the last row, write your own pair of equivalent expressions.

standard form ( $ax^2 + bx + c$ )	squared factor ( $(kx + m)^2$ )
$100x^2 + 80x + c$	
$36x^2 - 60x + c$	
$25x^2 + 40x + c$	
$0.25x^2 - 14x + c$	

- Solve each equation by completing the square:

$$25x^2 + 40x = -12$$

$$36x^2 - 60x + 10 = -6$$

## 14.4

## Putting Stars into Alignment

Here are three methods for solving  
 $3x^2 + 8x + 5 = 0$ .

Try to make sense of each method.

Method 1:

$$3x^2 + 8x + 5 = 0$$

$$(3x + 5)(x + 1) = 0$$

$$x = -\frac{5}{3} \quad \text{or} \quad x = -1$$

Method 2:

$$\begin{aligned} 3x^2 + 8x + 5 &= 0 \\ 9x^2 + 24x + 15 &= 0 \\ (3x)^2 + 8(3x) + 15 &= 0 \\ U^2 + 8U + 15 &= 0 \\ (U + 5)(U + 3) &= 0 \\ U = -5 \quad \text{or} \quad U &= -3 \\ 3x = -5 \quad \text{or} \quad 3x &= -3 \\ x = -\frac{5}{3} \quad \text{or} \quad x &= -1 \end{aligned}$$

Method 3:

$$\begin{aligned} 3x^2 + 8x + 5 &= 0 \\ 9x^2 + 24x + 15 &= 0 \\ 9x^2 + 24x + 16 &= 1 \\ (3x + 4)^2 &= 1 \\ 3x + 4 &= 1 \quad \text{or} \quad 3x + 4 = -1 \\ x = -1 \quad \text{or} \quad x &= -\frac{5}{3} \end{aligned}$$

Once you understand the methods, use each method at least one time to solve these equations.

1.  $5x^2 + 17x + 6 = 0$

2.  $6x^2 + 19x = -10$



$$3. \ 8x^2 - 33x + 4 = 0$$

$$4. \ 8x^2 - 26x = -21$$

$$5. \ 10x^2 + 37x = 36$$

$$6. \ 12x^2 + 20x - 77 = 0$$

## Are you ready for more?

Find the solutions to  $3x^2 - 6x + \frac{9}{4} = 0$ . Explain your reasoning.

## Lesson 14 Summary

In earlier lessons, we worked with perfect squares such as  $(x + 1)^2$  and  $(x - 5)(x - 5)$ . We learned that their equivalent expressions in standard form follow a predictable pattern:

- In general,  $(x + m)^2$  can be written as  $x^2 + 2mx + m^2$ .
- If a quadratic expression of the form  $ax^2 + bx + c$  is a perfect square, and the value of  $a$  is 1, then the value of  $b$  is  $2m$ , and the value of  $c$  is  $m^2$  for some value of  $m$ .

In this lesson, the variables in the factors being squared had coefficients other than 1, for example  $(3x + 1)^2$  and  $(2x - 5)(2x - 5)$ . Their equivalent expressions in standard form also followed the same pattern we saw earlier.

squared factor	standard form
$(3x + 1)^2$	$(3x)^2 + 2(3x)(1) + 1^2$ or $9x^2 + 6x + 1$
$(2x - 5)^2$	$(2x)^2 + 2(2x)(-5) + (-5)^2$ or $4x^2 - 20x + 25$

In general,  $(kx + m)^2$  can be written as:

$$(kx)^2 + 2(kx)(m) + m^2 \quad \text{or} \quad k^2x^2 + 2kxm + m^2$$

If a quadratic expression is of the form  $ax^2 + bx + c$ , then:

- The value of  $a$  is  $k^2$ .
- The value of  $b$  is  $2km$ .
- The value of  $c$  is  $m^2$ .

We can use this pattern to help us complete the square and solve equations when the squared term  $x^2$  has a coefficient other than 1—for example,  $16x^2 + 40x = 11$ .

What constant term  $c$  can we add to make the expression on the left of the equal sign a perfect square? And how do we write this expression as a squared factor?

- $a = 16$ , which is  $4^2$ , so  $k = 4$ , and the squared factor could be  $(4x + m)^2$ .
- 40 is equal to  $2(4m)$ , so  $2(4m) = 40$ , or  $8m = 40$ . This means that  $m = 5$ .
- If  $c$  is  $m^2$ , then  $c = 5^2$ , or  $c = 25$ .
- So the expression  $16x^2 + 40x + 25$  is a perfect square and is equivalent to  $(4x + 5)^2$ .

Let's solve the equation  $16x^2 + 40x = 11$  by completing the square!

$$\begin{aligned}16x^2 + 40x &= 11 \\16x^2 + 40x + 25 &= 11 + 25 \\(4x + 5)^2 &= 36\end{aligned}$$

$$\begin{aligned}4x + 5 &= 6 & \text{or} & \quad 4x + 5 = -6 \\4x &= 1 & \text{or} & \quad 4x = -11 \\x &= \frac{1}{4} & \text{or} & \quad x = -\frac{11}{4}\end{aligned}$$