



# Efficiently Solving Inequalities

Let's solve more complicated inequalities.

## 15.1 Lots of Negatives

Here is an inequality:  $-x \geq -4$

- Without testing any values, predict what the solutions to this inequality will look like on the number line.
  - Where will the starting (or boundary) point be?
  - Will the circle at that point be filled or unfilled?
  - Will the shading go to the right or to the left of that point?
- Test each value to see whether it is a solution to the inequality  $-x \geq -4$ .
  - 3
  - 3
  - 4
  - 4
  - 4.001
  - 4.001
- Graph all possible solutions to the inequality on the number line:



## 15.2 Inequalities with Tables

1. Let's investigate the inequality  $x - 3 > -2$ .

|         |    |    |    |    |   |   |    |   |   |
|---------|----|----|----|----|---|---|----|---|---|
| $x$     | -4 | -3 | -2 | -1 | 0 | 1 | 2  | 3 | 4 |
| $x - 3$ | -7 | -6 |    | -4 |   |   | -1 |   | 1 |

- Complete the table.
- For which values of  $x$  is it true that  $x - 3 = -2$ ?
- For which values of  $x$  is it true that  $x - 3 > -2$ ?
- Graph the solutions to  $x - 3 > -2$  on the number line:



2. Here is an inequality:  $2x < 6$ .

- Predict which values of  $x$  will make the inequality  $2x < 6$  true.
- Complete the table. Does it match your prediction?

|      |    |    |    |    |   |   |   |   |   |
|------|----|----|----|----|---|---|---|---|---|
| $x$  | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| $2x$ |    |    |    |    |   |   |   |   |   |

- Graph the solutions to  $2x < 6$  on the number line:



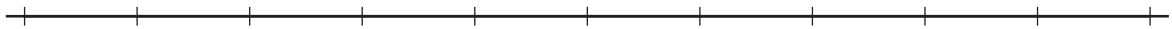
3. Here is an inequality:  $-2x < 6$ .

a. Predict which values of  $x$  will make the inequality  $-2x < 6$  true.

b. Complete the table. Does it match your prediction?

|       |    |    |    |    |   |   |   |   |   |
|-------|----|----|----|----|---|---|---|---|---|
| $x$   | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| $-2x$ |    |    |    |    |   |   |   |   |   |

c. Graph the solutions to  $-2x < 6$  on the number line:



4. How are the solutions to  $2x < 6$  different from the solutions to  $-2x < 6$ ?

## 15.3 Which Side Shows the Solutions?

1. Let's investigate  $-4x + 5 \geq 25$ .

a. Solve  $-4x + 5 = 25$ .

b. Is  $-4x + 5 \geq 25$  true when  $x$  is 0? What about when  $x$  is 7? What about when  $x$  is -7?

c. Graph the solutions to  $-4x + 5 \geq 25$  on the number line.



2. Let's investigate  $\frac{x}{3} + 7 < 19$ .

a. Solve  $\frac{x}{3} + 7 = 19$ .

b. Is  $\frac{x}{3} + 7 < 19$  true when  $x$  is 0?

c. Graph the solutions to  $\frac{x}{3} + 7 < 19$  on the number line.



3. Solve the inequality  $3(x - 4) > 16.5$ , and graph the solutions on the number line.



4. Solve the inequality  $-2\left(x + \frac{3}{2}\right) \leq 6$ , and graph the solutions on the number line.



## Are you ready for more?

Write at least three different inequalities whose solution is  $x > -10$ . Find one inequality with  $x$  on the left side that uses a  $<$ .

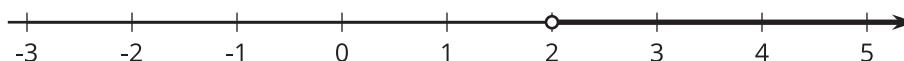
## Lesson 15 Summary

Here is an inequality:  $3(10 - 2x) < 18$ . The solution set for this inequality is all the values that can be used in place of  $x$  to make the inequality true. Each solution is one value that makes the inequality true.

In order to solve this inequality, we can first solve the related equation  $3(10 - 2x) = 18$  to get the solution  $x = 2$ . That means 2 is the boundary between values of  $x$  that make the inequality true and values that make the inequality false.

To solve the inequality, we can check numbers greater than 2 and less than 2 and see which ones make the inequality true.

Let's check a number that is greater than 2:  $x = 5$ . Replacing  $x$  with 5 in the inequality, we get  $3(10 - 2 \cdot 5) < 18$  or just  $0 < 18$ . This is true, so  $x = 5$  is a solution. This means that all values greater than 2 make the inequality true. We can represent the solutions as  $x > 2$  and also represent the solutions on a number line:



Notice that 2 itself is not a solution because it's the value of  $x$  that makes  $3(10 - 2x)$  *equal* to 18, and so it does not make  $3(10 - 2x) < 18$  true.

For confirmation that we found the correct solution, we can also test a value that is less than 2. If we test  $x = 0$ , we get  $3(10 - 2 \cdot 0) < 18$  or just  $30 < 18$ . This is false, so  $x = 0$  and all values of  $x$  that are less than 2 are not solutions.