Scope and Sequence for Grade 7

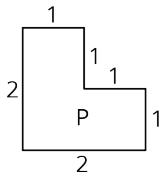
IM Grade 7 begins with students studying scale drawings, an engaging geometric topic that sets the stage for the subsequent work on proportional relationships in the following three units. Students also have opportunities to build fluency with IM Grade 6 arithmetic. They work with proportional relationships represented by tables, equations, and graphs. Geometry and proportional relationships are interwoven in the third unit, when the important proportional relationship between a circle's circumference and its diameter is studied. Then students work with percent increase and percent decrease.

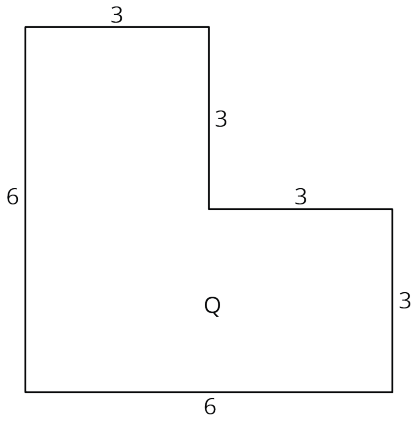
By the fifth unit, on operations with rational numbers, students have had time to brush up on and solidify their understanding of, and skill in, IM Grade 6 arithmetic. At this point, the emphasis becomes the role of the properties of operations in determining the rules for operating with negative numbers. This is a natural lead-in to the work on solving equations and simplifying expressions in the next unit. Students then put their arithmetical and algebraic skills to work in the last two units: on angles, triangles, and prisms, and on probability and sampling.

The final unit of the course is optional. The lessons provide students with additional opportunities to integrate and apply various ideas from the course to solve real-world and mathematical problems.

Unit 1: Scale Drawings

In this unit, students study scaled copies of plane figures and scale drawings of real-world objects. Students learn that all lengths in a scaled copy are the result of multiplying the original lengths by a scale factor. Also, the angle measures in a scaled copy are the same as in the original figure.





This work builds on what students learned in previous grades about measuring lengths, areas, and angles. This unit provides a geometric context to preview the type of reasoning that students will use with proportional relationships later in grade 7. It also lays the foundation for grade 8 work on dilations and similarity.

Students begin the unit by looking at copies of a picture and describing what differentiates scaled and non-scaled copies. They calculate scale factors and draw scaled copies of figures. Note that the study of scaled copies is limited to pairs of figures that have the same orientation — in other words, they are not rotations or reflections of each other. In grade 8, students will extend their knowledge of scaled copies when they study translations, rotations, reflections, and dilations.

Next, students study scale drawings. They see that the principles and strategies that they used to reason about scaled copies of figures can also be used with scale drawings. They use scale drawings to calculate actual lengths and areas, and they create scale drawings.

A note about the geometry toolkit:

In the unit, several lesson plans suggest that each student have access to a *geometry toolkit*. Each toolkit contains tracing paper, graph paper, colored pencils, scissors, a centimeter ruler, a protractor (clear protractors with no holes

that show radial lines are recommended), and an index card to use as a straightedge or to mark right angles. Providing students with these toolkits gives opportunities for students to develop abilities to select appropriate tools and use them strategically to solve problems (MP5). Note that even students in a digitally enhanced classroom should have access to such tools. Applets and simulations should be considered additions to their toolkits, not replacements for physical tools.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as representing, generalizing, and explaining. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Represent

- A scaled copy for a given scale factor (Lessons 3 and 5).
- Distances using different scales (Lesson 11).
- Relevant features of a classroom with a scale drawing (Lesson 13).

Generalize

- About corresponding distances and angles in scaled copies (Lesson 4).
- About scale factors greater than, less than, and equal to 1 (Lesson 5).
- About scale factors and area (Lessons 6 and 10).
- About scale factors with and without units (Lesson 12).

Explain

- How to use scale drawings to find actual distances (Lessons 7 and 11).
- How to use scale drawings to find actual distances, speed, and elapsed time (Lesson 8).
- How to use scale drawings to find actual areas (Lesson 12).

In addition, students are expected to describe features of scaled copies, justify and critique reasoning about scaled copies, and compare how different scales affect drawings. Over the course of the unit, teachers can support students' mathematical understandings by amplifying (not simplifying) language used for all of these purposes as students demonstrate and develop ideas.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
7.1.1	**scaled copy** original polygon	
7.1.2	**corresponding** **scale factor** figure segment	
7.1.4	quadrilateral measurement distance	**corresponding** **scale factor** original
7.1.5	**reciprocal**	
7.1.6	area one-dimensional two-dimensional	squared
7.1.7	**scale drawing** **scale** represent actual three-dimensional	**scaled copy**
7.1.8	estimate travel constant speed	**scale**
7.1.9	floor plan	
7.1.10	appropriate dimension	actual represent
7.1.11	scale without units ___ to ___	**scale drawing**
7.1.12	equivalent scales	___ to ___

Section A: Scaled Copies

- Lesson 1: What Are Scaled Copies?
- Lesson 2: Corresponding Parts and Scale Factors
- Lesson 3: Making Scaled Copies
- Lesson 4: Scaled Relationships
- Lesson 5: The Size of the Scale Factor
- Lesson 6: Scaling and Area

Section B: Scale Drawings

- Lesson 7: Scale Drawings
- Lesson 8: Scale Drawings and Maps
- Lesson 9: Creating Scale Drawings
- Lesson 10: Changing Scales in Scale Drawings
- Lesson 11: Scales without Units
- Lesson 12: Units in Scale Drawings

Section C: Let's Put It to Work

- Lesson 13: Draw It to Scale

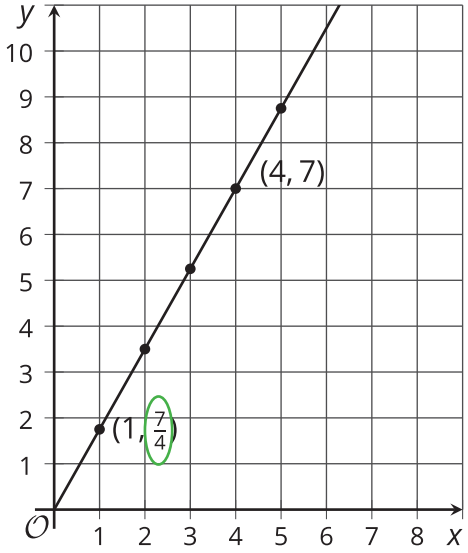
Unit 2: Introducing Proportional Relationships

In this unit, students develop the idea of a proportional relationship. They work with proportional relationships that are represented in tables, as equations, and on graphs. This builds on grade 6 work with equivalent ratios and helps prepare students for the study of linear functions in grade 8.

Students begin by looking at tables. In a table of equivalent ratios, a multiplicative relationship between a pair of rows is given by a scale factor, while the multiplicative relationship between the columns is given by a unit rate. Students learn that the relationship between pairs of values in the two columns is called a "proportional relationship," and the unit rate that describes this relationship is called a "constant of proportionality."

Next, students use equations of the form $y = kx$ to represent proportional relationships and solve problems. They determine whether given tables and equations could represent a proportional relationship.

Then students investigate graphs of proportional relationships. They recognize that the graph of a proportional relationship is a straight line through $(0, 0)$. They interpret points on the graph, including the point $(1, k)$. Here is an example of a graph, an equation, and a table that all represent the same proportional relationship.



$y = \frac{7}{4}x$

x	y
0	0
1	$\frac{7}{4}$
2	$\frac{7}{2}$
3	$\frac{21}{4}$
4	7

By the end of the unit, students should be comfortable working with common contexts associated with proportional relationships (such as constant speed, unit pricing, and measurement conversions) and be able to determine whether or not a relationship is proportional. In a later unit, students will apply proportional reasoning to solve multi-step problems and to calculate more complex rates.

A note on using the terms "ratio," "proportional relationship," and "unit rate":

In these materials, the term "ratio" is used to mean a type of association between two or more quantities. A quantity is a measurement that can be specified by a number and a unit, for example 4 oranges, 4 centimeters, or “my height in feet.” A proportional relationship is a collection of equivalent ratios.

A unit rate is the numerical part of a rate per 1 unit, for example, the 6 in 6 miles per hour. The fractions $\frac{a}{b}$ and $\frac{b}{a}$ are never called ratios. The fractions $\frac{a}{b}$ and $\frac{b}{a}$ are identified as “unit rates” for the ratio $a : b$. In high school—after the study of ratios, rates, and proportional relationships—students discard the term “unit rate” and start referring to a to b, $a : b$, and $\frac{a}{b}$ as “ratios.”

In grades 6–8, students write rates without abbreviated units, for example as “3 miles per hour” or “3 miles in every 1 hour.” Use of notation for derived units such as $\frac{\text{mi}}{\text{hr}}$ waits for high school—except for the special cases of area and volume.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as comparing, interpreting, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Compare

- Drink mixtures and figures (Lesson 1).
- Approaches to solving problems involving proportional relationships (Lesson 6).
- Proportional relationships with nonproportional relationships (Lesson 8).
- Tables, descriptions, and graphs representing the same situations (Lesson 10).
- Graphs of proportional relationships (Lesson 12).

Interpret

- Representations showing equivalent ratios (Lesson 1).
- Tables showing equivalent ratios (Lesson 2).
- Situations involving proportional relationships (Lesson 6 and 9).
- How a graph represents features of a situation (Lesson 11).

Generalize

- About proportional relationships (Lesson 4).
- About equations that represent proportional relationships (Lesson 5).
- About how a constant of proportionality is represented by graphs and tables (Lesson 13).

In addition, students are expected to describe proportional relationships and constants of proportionality, explain how to determine whether or not a relationship is proportional and how to compare and represent situations with different constants of proportionality, justify whether or not a relationship is proportional, and represent proportional and nonproportional relationships in multiple ways.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students’ use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
7.2.1	equivalent ratios	
7.2.2	**constant of proportionality** **proportional relationship** value	equivalent ratios row column
7.2.3	___ is proportional to ___ relate constant	**reciprocal** per
7.2.4	equation quotient	___ is proportional to ___
7.2.5	steady situation	
7.2.6		equation quotient
7.2.7		**constant of proportionality** **proportional relationship**
7.2.8		constant
7.2.10	**origin** coordinate plane plot	
7.2.11	quantity axes coordinates	
7.2.13	x-coordinate y-coordinate	**origin**
7.2.14		axes
7.2.15	reasonable	

Section A: Representing Proportional Relationships with Tables

- Lesson 1: One of These Things Is Not Like the Others
- Lesson 2: Introducing Proportional Relationships with Tables
- Lesson 3: More about Constant of Proportionality

Section B: Representing Proportional Relationships with Equations

- Lesson 4: Proportional Relationships and Equations
- Lesson 5: Two Equations for Each Relationship

- Lesson 6: Writing Equations to Represent Relationships

Section C: Comparing Proportional and Nonproportional Relationships

- Lesson 7: Comparing Relationships with Tables
- Lesson 8: Comparing Relationships with Equations
- Lesson 9: Solving Problems about Proportional Relationships

Section D: Representing Proportional Relationships with Graphs

- Lesson 10: Introducing Graphs of Proportional Relationships
- Lesson 11: Interpreting Graphs of Proportional Relationships
- Lesson 12: Using Graphs to Compare Relationships
- Lesson 13: Two Graphs for Each Relationship

Section E: Let's Put It to Work

- Lesson 14: Four Representations
- Lesson 15: Using Water Efficiently

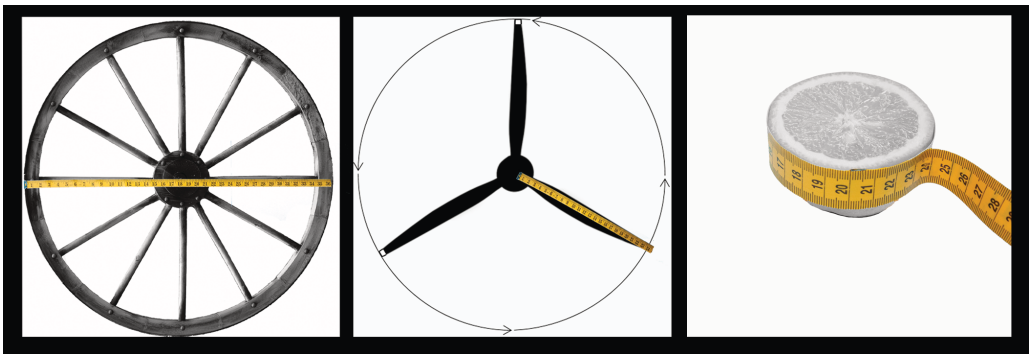
Unit 3: Measuring Circles

In this unit, students apply their knowledge of proportional relationships to the context of measuring circles. They learn the relationships between radius, diameter, circumference, and area of circles and use these relationships to solve problems. This builds on students' work from previous grades with perimeter and area of polygons. Students will build on this work in grade 8 when they study the volume of spheres, cylinders, and cones.

The unit begins with activities designed to help build up students' vocabulary for describing circles more precisely. The terms "center," "radius," "diameter," and "circumference" are introduced. Then students investigate the relationship between circumference and diameter and see that it is a proportional relationship. They apply this relationship to solve problems.

Next, students explore the area of circular regions. They see an informal derivation that shows where the formula $A = \pi r^2$ comes from and then use this formula to solve problems. Finally, students solve problems that require deciding whether the situation relates to the circumference or area of a circle.

The first section of this unit, in which students recognize and apply proportional relationships involving circumference, serves as a bridge between the foundational work with proportional relationships in the previous unit and the more advanced applications in the following unit. The remaining sections of this unit, which deal with the area of circles, are preparation for the continued geometry work students will do later in this course.



A note on using the term "circle":

Strictly speaking, a circle is one-dimensional. It is the boundary of a two-dimensional region, rather than the region itself. The circular region is called a "disk." Because students are not yet expected to make this distinction, these materials refer to both disks and the boundaries of disks as "circles," using illustrations to eliminate ambiguity.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as generalizing, justifying, and interpreting. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Generalize

- About categories for sorting circles (Lesson 2).
- About the relationship between circumference and diameter (Lesson 3).
- About circumference and rotation (Lesson 5).
- About the relationship between the radius and the area of a circle (Lesson 8).

Justify

- Reasoning about circumference and perimeter (Lesson 4).
- Estimates for the areas of circles (Lesson 7).
- Reasoning about areas of curved figures (Lesson 9).
- Reasoning about the cost of stained-glass windows (Lesson 11).

Interpret

- Situations involving circles (Lessons 5 and 8).
- Floor plans and maps (Lesson 6).
- Situations involving circumference and area (Lesson 10).

In addition, students are expected to critique reasoning about circles and circle measurements, explain reasoning, including about different approximations of pi, and describe features of graphs and of deconstructed circles.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
7.3.1	relationship perimeter	
7.3.2	**radius** **diameter** **circumference** center (of a circle)	**circle**
7.3.3	**pi**	
7.3.4	half-circle rotation approximation	
7.3.5		**diameter** **circumference** **pi** travel
7.3.6		approximate estimate
7.3.7	**area of a circle**	
7.3.8	**squared** formula	**radius** **area of a circle**
7.3.9	in terms of π	
7.3.10		**squared** center (of a circle) formula
7.3.11	design	

Section A: Circumference of a Circle

- Lesson 1: How Well Can You Measure?
- Lesson 2: Exploring Circles
- Lesson 3: Exploring Circumference
- Lesson 4: Applying Circumference
- Lesson 5: Circumference and Wheels

Section B: Area of a Circle

- Lesson 6: Estimating Areas
- Lesson 7: Exploring the Area of a Circle

- Lesson 8: Relating Area to Circumference
- Lesson 9: Applying Area of Circles

Section C: Let's Put It to Work

- Lesson 10: Distinguishing Circumference and Area
- Lesson 11: Stained-Glass Windows

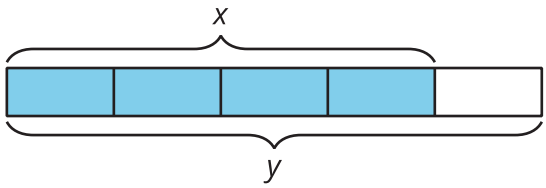
Unit 4: Proportional Relationships and Percentages

In this unit, students deepen their understanding of proportional relationships and percentages. They solve multi-step problems and work with situations that involve fractional amounts. This builds on the work students did in grade 6 with ratios, rates, and percentages as well as previous units in grade 7 with proportional relationships. Students will build on this work in high school with exponential functions representing compounded percent increase and decrease.

Students begin the unit by revisiting constant rates, but this time the given values are fractional amounts. To determine the unit rate for the situation, students must compute the quotient of two fractions. Students also make sense of situations where an increase or decrease is expressed as a fraction of the initial amount. They create diagrams and apply the distributive property to generate expressions that represent these situations. They also use long division to write fractions as decimals, including their first introduction to repeating decimals.

Next, students make sense of situations where an increase or decrease is expressed as a percentage of the initial amount. They continue creating diagrams and writing equations to represent the situations. They solve for any one of the three quantities—the initial amount, the final amount, or the percentage of the change—given the other two quantities. They also reason about fractional percentages.

Then students apply percent increase and decrease to solve problems in a variety of real-world situations, such as tax, tip, interest, markup, discount, depreciation, and commission. Lastly, students make sense of situations where the difference between a correct measurement and an incorrect measurement is expressed as a percentage of the correct amount.



$y = x + \frac{1}{4}x$

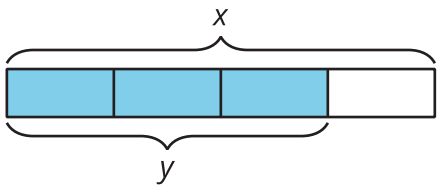
$y = (1 + \frac{1}{4})x$

$y = \frac{5}{4}x$

$y = (1 + 0.25)x$

$y = 1.25x$

"a 25% increase"



$y = x - \frac{1}{4}x$

$y = (1 - \frac{1}{4})x$

$y = \frac{3}{4}x$

$y = (1 - 0.25)x$

$y = 0.75x$

"a 25% decrease"

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, explaining, and representing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret

- Situations involving constant speed (Lesson 2).

- Concrete problems involving percent increase and decrease (Lesson 7).
- Problems involving sales tax and tip (Lesson 10).
- Concrete situations involving percent error (Lesson 14).

Explain

- How to solve concrete and abstract problems involving an amount plus (or minus) a fraction of that amount (Lesson 4).
- How to solve percent change problems (Lesson 6).
- Strategies for solving percent problems with fractional percentages (Lesson 9).
- How to measure lengths and interpret measurement error (Lesson 13).
- Strategies for solving percent error problems (Lesson 14).

Represent

- Situations involving percent increase and decrease (Lessons 8 and 15).
- Situations from the news involving percent change (Lesson 16).

In addition, students are expected to compare measurements, scale factors, and decimal and fraction representations, compare representations of an increase (or decrease) of an amount by a fraction or decimal, generalize about using constants of proportionality to solve problems efficiently and about relationships with percent increase and decrease, and justify why specific information is needed to solve percent change problems.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
7.4.1	percentage	
7.4.2		**unit rate**
7.4.4	(a fraction) more than (a fraction) less than initial/original amount final/new amount	**tape diagram** distributive property
7.4.5	**repeating decimal** **long division** decimal representation	
7.4.6	**percent increase** **percent decrease**	(a fraction) more than (a fraction) less than
7.4.7	discount	initial/original amount final/new amount
7.4.10	sales tax tax rate tip	**percent increase**
7.4.11	interest commission markup markdown	**percent decrease**
7.4.12		percentage discount
7.4.13	**measurement error**	
7.4.14	**percent error**	

Section A: Proportional Relationships with Fractions

- Lesson 1: Lots of Flags
- Lesson 2: Ratios and Rates with Fractions
- Lesson 3: Revisiting Proportional Relationships
- Lesson 4: More than That, Less than That
- Lesson 5: Say It with Decimals

Section B: Percent Increase and Decrease

- Lesson 6: Increasing and Decreasing
- Lesson 7: One Hundred Percent

- Lesson 8: Percent Increase and Decrease with Equations
- Lesson 9: Part of a Percent

Section C: Applying Percentages

- Lesson 10: Tax and Tip
- Lesson 11: Percentage Contexts
- Lesson 12: Solving Multi-step Percentage Problems
- Lesson 13: Measurement Error
- Lesson 14: Percent Error

Section D: Let's Put It to Work

- Lesson 15: Changes on the Earth
- Lesson 16: Posing Percentage Problems

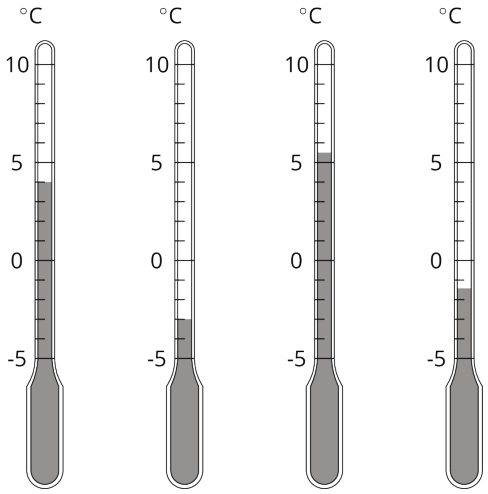
Unit 5: Rational Number Arithmetic

In this unit, students perform operations on rational numbers, which are all numbers that can be written as a positive or negative fraction. This builds on grade 6 work with interpreting, comparing, and plotting rational numbers. It prepares students for a later unit when they will solve equations of the form $px + q = r$ or $p(x + q) = r$, where p, q, and r are rational numbers.

Students begin by revisiting how signed numbers are used to represent quantities above and below a reference point, such as measurements of temperature and elevation. They use tables and number line diagrams to represent changes in temperature or elevation. They extend addition and subtraction from fractions to all rational numbers. And they see that $a - b$ is equivalent to $a + (\text{-}b)$.

Next, students examine multiplication and division. They work with constant velocity, which is a signed number that indicates direction and speed. This allows products of signed numbers to be interpreted in terms of position, direction of movement, and time before or after a specific point. Students use the relationship between multiplication and division to understand how division extends to rational numbers.

Then students work with expressions that use the four operations on rational numbers. They also solve problems that involve interpreting negative numbers in context. They solve linear equations of the form $p + x = q$ or $px = q$, where p and q rational numbers. The focus of these lessons is representing situations with equations and what it means for a number to be a solution for an equation, rather than methods for solving equations. Such methods are the focus of a later unit.



A note on using the terms "expression," "equation," and "signed number":

In these materials, an *expression* is built from numbers, variables, operation symbols (+, −, ·, ÷), parentheses, and exponents. (Exponents—in particular, negative exponents—are not a focus of this unit. Students work with integer exponents in grade 8 and noninteger exponents in high school.) An *equation* is a statement that two expressions are equal, thus it always has an equal sign. *Signed numbers* include all rational numbers, written as decimals or in the form $\frac{a}{b}$.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, representing, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret

- Situations involving signed numbers (throughout unit).
- Tables with signed numbers (Lesson 3).
- Bank statements with signed numbers (Lesson 4).

Represent

- Addition of signed numbers on a number line (Lesson 2).
- Situations involving signed numbers (Lessons 3 and 11).
- Changes in elevation (Lesson 6).
- Position, speed, and direction (Lesson 8).

Generalize

- About subtracting and adding signed numbers (Lesson 5).
- About differences and magnitude (Lesson 6).
- About multiplying negative numbers (Lesson 9).
- About additive and multiplicative inverses (Lesson 15).

In addition, students are expected to justify reasoning about distances on a number line and about negative numbers, account balances, and debt. Students are also expected to explain how to determine changes in temperature, how to find information using inverses, and how to model situations involving signed numbers.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
7.5.1	degrees Celsius vertical elevation sea level	**positive number** **negative number**
7.5.2	signed numbers	temperature number line
7.5.3	**opposite** sum expression	
7.5.4	**deposit** **withdrawal** account balance debt	
7.5.6	difference	distance
7.5.7		absolute value x-coordinate y-coordinate
7.5.8	velocity	
7.5.11	**solution (to an equation)** factor	
7.5.13	rational number variable	sum difference
7.5.15	additive inverse multiplicative inverse	**opposite** **solution (to an equation)**
7.5.17		increase decrease

Section A: Adding and Subtracting Rational Numbers

- Lesson 1: Interpreting Negative Numbers
- Lesson 2: Changing Temperatures
- Lesson 3: Changing Elevation
- Lesson 4: Money and Debts
- Lesson 5: Representing Subtraction
- Lesson 6: Finding Differences
- Lesson 7: Adding and Subtracting to Solve Problems

Section B: Multiplying and Dividing Rational Numbers

- Lesson 8: Multiplying Rational Numbers (Part 1)
- Lesson 9: Multiplying Rational Numbers (Part 2)
- Lesson 10: Multiply!
- Lesson 11: Dividing Rational Numbers
- Lesson 12: Negative Rates

Section C: Four Operations with Rational Numbers

- Lesson 13: Expressions with Rational Numbers
- Lesson 14: Solving Problems with Rational Numbers
- Lesson 15: Solving Equations with Rational Numbers
- Lesson 16: Representing Contexts with Equations

Section D: Let's Put It to Work

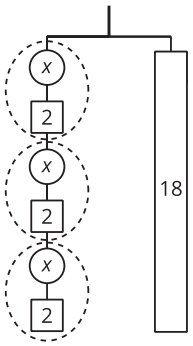
- Lesson 17: The Stock Market

Unit 6: Expressions, Equations, and Inequalities

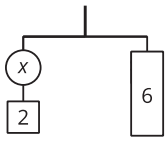
In this unit, students deepen their algebraic reasoning as they write and solve equations of the forms $px + q = r$ and $p(x + q) = r$ and inequalities of the forms $px + q > r$ and $p(x + q) < r$. Students also work with equivalent expressions that are more complex than what they have seen previously. This builds on grade 6 work with equations of the form $p + x = q$ or $px = q$ and with simpler equivalent expressions. Students will build on this work in grade 8 when they solve equations that have a variable on both sides of the equal sign and when they work with systems of equations.

Students begin the unit by making sense of situations that involve both multiplication and addition. They represent such situations with tape diagrams and with equations. They see that different diagrams and equations can represent the same situation, and they use diagrams to find solutions to equations.

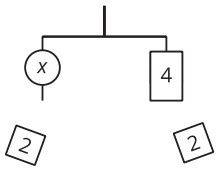
Next, students consider hanger diagrams as another way to represent equations. The diagrams help students understand solving equations in terms of "doing the same thing to each side of the equation." Students examine different pathways for solving the same equation and consider whether one method is more efficient than another.



$3(x + 2) = 18$



$x + 2 = 6$



$x = 4$

Then students apply what they have learned about equations to inequalities. They write inequalities to represent situations and solve inequalities by reasoning about the related equation. The inequality symbols $\geq$ and $\leq$ are introduced.

Lastly, students work with equivalent linear expressions that are more complex due to having more terms, more parentheses, and negative rational numbers. Students use properties of operations to justify why the expressions are

equivalent.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as comparing, explaining, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Compare

- Stories with corresponding tape diagrams (Lesson 2).
- Tape diagrams with corresponding equations (Lesson 3).
- Hanger diagrams and equations (Lesson 7).
- Solution pathways (especially Lesson 10).
- Descriptions of situations with corresponding inequalities (Lesson 16).

Explain

- Strategies for using hanger diagrams to solve equations (Lesson 8).
- Different strategies for solving equations (Lesson 9) and inequalities (Lesson 14).
- Reasoning about situations, tape diagrams, and equations (Lesson 12).
- Strategies for identifying and writing equivalent expressions (Lesson 21).

Justify

- Reasoning about inequalities (Lesson 13).
- Reasoning about solutions to inequalities (Lesson 15).
- The need for specific information in order to write and solve inequalities (Lesson 17).
- Reasoning about the distributive property (Lesson 19).
- Whether different sequences of calculations give the same result (Lesson 22).

In addition, students are expected to interpret solutions to equations, interpret and represent nonproportional situations with constant rates of change, represent nonproportional situations using tape diagrams, describe the structure of equations and tape diagrams, critique reasoning of peers about expressions and corresponding diagrams, critique reasoning about solving equations, critique reasoning about equivalent expressions, and generalize about solving equations and about when expressions are equivalent.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
7.6.2	unknown amount	
7.6.3	**equivalent expressions** commutative (property)	
7.6.4		unknown amount relationship
7.6.6		**variable**
7.6.7	balanced hanger each side (of an equation)	
7.6.8		**equivalent expression** each side (of an equation)
7.6.9		operation solve
7.6.10	distribute substitute	
7.6.13	inequality less than or equal to greater than or equal to open / closed circle	less than greater than
7.6.14	**solution to an inequality** boundary direction (of an inequality)	less than or equal to greater than or equal to substitute
7.6.15		open / closed circle
7.6.16		**solution to an inequality**
7.6.17		inequality
7.6.18	**term**	
7.6.19	**factor (an expression)** **expand (an expression)**	
7.6.20	combine like terms	**term** commutative (property)
7.6.21		distribute

Section A: Representing Situations of the Form $px + q = r$ and $p(x + q) = r$

- Lesson 1: Relationships between Quantities

- Lesson 2: Reasoning about Contexts with Tape Diagrams
- Lesson 3: Reasoning about Equations with Tape Diagrams
- Lesson 4: Reasoning about Equations and Tape Diagrams (Part 1)
- Lesson 5: Reasoning about Equations and Tape Diagrams (Part 2)
- Lesson 6: Distinguishing between Two Types of Situations

Section B: Solving Equations of the Form $px + q = r$ and $p(x + q) = r$ and Problems That Lead to Those Equations

- Lesson 7: Reasoning about Solving Equations (Part 1)
- Lesson 8: Reasoning about Solving Equations (Part 2)
- Lesson 9: Dealing with Negative Numbers
- Lesson 10: Different Options for Solving One Equation
- Lesson 11: Using Equations to Solve Problems
- Lesson 12: Solving Problems about Percent Increase or Decrease

Section C: Inequalities

- Lesson 13: Reintroducing Inequalities
- Lesson 14: Finding Solutions to Inequalities in Context
- Lesson 15: Efficiently Solving Inequalities
- Lesson 16: Interpreting Inequalities
- Lesson 17: Modeling with Inequalities

Section D: Writing Equivalent Expressions

- Lesson 18: Subtraction in Equivalent Expressions
- Lesson 19: Expanding and Factoring
- Lesson 20: Combining Like Terms (Part 1)
- Lesson 21: Combining Like Terms (Part 2)

Section E: Let's Put It to Work

- Lesson 22: Applications of Expressions

Unit 7: Angles, Triangles, and Prisms

In this unit, students investigate whether sets of angle and side length measurements determine unique triangles or multiple triangles, or fail to determine triangles. Students also study and apply angle relationships, learning to understand and use the terms "complementary," "supplementary," "vertical angles," and "unique." The work gives them practice working with rational numbers and equations for angle relationships. Students analyze and describe cross-sections of prisms, pyramids, and polyhedra. They understand and use the formula for the volume of a right rectangular prism and solve problems involving area, surface area, and volume. Students should have access to their geometry toolkits so that they have an opportunity to select and use appropriate tools strategically.

Note: It is not expected that students memorize which conditions result in a unique triangle, an impossible-to-create triangle, or multiple possible triangles. Understanding that, for example, side-side-side (SSS) information results in zero

or exactly one triangle will be explored in high school geometry. At this level, students should attempt to draw triangles with the given information and notice that there is only one way to do it (or that it is impossible to do).

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as critiquing, explaining, interpreting, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Critique

- Reasoning about measuring angles (Lesson 1).
- Reasoning about decomposition of prisms (Lesson 13).
- Reasoning about surface area of prisms (Lesson 14).

Explain

- How to measure angles (Lesson 2).
- How to find unknown angle measurements (Lessons 4 and 5).
- How to find the volume of prisms (Lessons 12 and 13).
- How to find the surface area of prisms (Lesson 14).

Interpret

- Situations involving intersecting lines in order to form a conjecture (Lesson 3).
- Which information is relevant to answer questions (Lesson 4).
- Equations representing angle measurements (Lesson 5).
- Situations involving volume and surface area (Lesson 15 and 16).

Justify

- Whether or not shapes are identical copies (Lesson 6).
- Whether or not measurements determine identical copies (Lesson 9).
- Whether or not measurements determine unique triangles (Lesson 10).

In addition, students are expected to use language to compare angle measurements, compare triangles in a set, compare cross-sections of figures, describe characteristics of pattern blocks, describe positioning and movement of side lengths and angles, and describe cross-sections of prisms and pyramids. Students also have opportunities to generalize about patterns of angle measurements, about categories for unique triangles, and about categories for cross-sections.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
7.7.1	**straight angle** **adjacent angles** degree	**right angle**
7.7.2	**supplementary** **complementary** angle measure protractor	**measurement error** degrees
7.7.3	**vertical angles** intersect vertex (of an angle)	
7.7.4		**supplementary** **vertical angles**
7.7.5	perpendicular	**complementary**
7.7.6	identical copy condition	angle measurement side length quadrilateral
7.7.7	compass different triangle	intersect identical copy segment
7.7.8		condition different triangle
7.7.9	unique triangle parallel	
7.7.10		protractor compass
7.7.11	**cross-section** **base (of a prism or pyramid)** vertex (of a pyramid) face	**prism** **pyramid** perpendicular parallel
7.7.12		**volume** **cross-section** **base (of a prism or pyramid)**
7.7.14		face perimeter
7.7.15		**surface area**

Section A: Angle Relationships

- Lesson 1: Relationships of Angles
- Lesson 2: Adjacent Angles
- Lesson 3: Nonadjacent Angles
- Lesson 4: Solving for Unknown Angles
- Lesson 5: Using Equations to Solve for Unknown Angles

Section B: Drawing Polygons with Given Conditions

- Lesson 6: Building Polygons (Part 1)
- Lesson 7: Building Polygons (Part 2)
- Lesson 8: Triangles with 3 Common Measures
- Lesson 9: Drawing Triangles (Part 1)
- Lesson 10: Drawing Triangles (Part 2)

Section C: Solid Geometry

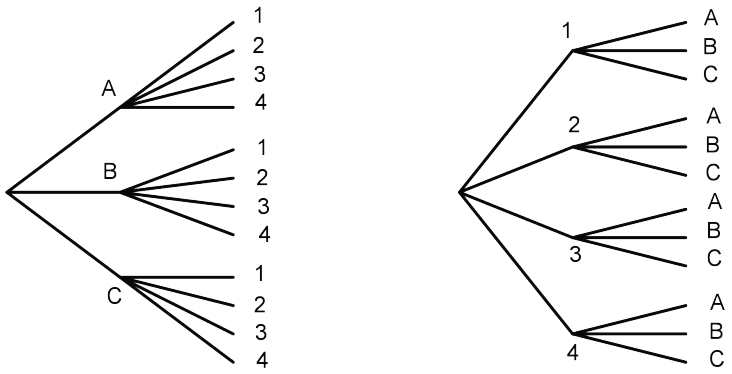
- Lesson 11: Slicing Solids
- Lesson 12: Volume of Right Prisms
- Lesson 13: Decomposing Bases for Area
- Lesson 14: Surface Area of Right Prisms
- Lesson 15: Distinguishing Volume and Surface Area
- Lesson 16: Applying Volume and Surface Area

Section D: Let's Put It to Work

- Lesson 17: Building Prisms

Unit 8: Probability and Sampling

In this unit, students work with probability and sampling. They use their understanding of basic chance experiments to quantify how likely events are to happen and develop a working understanding of probability. Then they design and use simulations to further understand probability as the frequency of the event occurring when repeating an experiment many times. Students represent sample spaces using tables, tree diagrams, and lists, and use the number of outcomes in a sample space to calculate an expected probability.



Next, students examine different ways to collect data from samples within a population to understand why random selection is useful. Then students generate samples and estimate information about the population from sample data. Finally, students compare two groups by examining the measures of center and measures of variability calculated from sample data representing each group.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as describing, explaining, justifying, and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe

- Observations and predictions during a game (Lesson 1).
- Patterns observed in repeated experiments (Lesson 4).
- Chance experiments to model situations (Lessons 6 and 7).
- A simulation used to model a situation (Lesson 10).
- Observations about data sets (Lessons 11 and 17).

Explain

- Predictions (Lesson 2).
- How to determine which events are more likely (Lesson 3).
- Possible differences in experimental and theoretical probability (Lesson 5).
- How to use simulations to estimate probability (Lesson 7).
- How to use a simulation to answer questions about the situation (Lesson 10).

Justify

- Whether situations are surprising and possible (Lesson 4).
- Which samples are or are not representative of a larger population (Lesson 13).
- Which samples correspond with each show, which show is most appropriate for a commercial, and whether a movie is eligible for an award (Lesson 15).
- Reasoning about samples and populations (Lesson 16).
- Whether or not differences between samples are meaningful (Lesson 18, 19, and 20).

Compare

- Sample spaces and probability of outcomes for different spinners (Lesson 5).
- Methods for writing sample spaces (Lesson 8).
- Heights of two groups (Lesson 11).
- Measures of center with samples (Lesson 13).
- Sampling methods (Lesson 14).
- Populations based on samples (Lessons 18 and 20).

In addition, students are expected to critique predictions about the mean of random samples and generalize about sample spaces, predictions, sampling, and fairness. Students also have opportunities to use language to represent data from repeated experiments, represent probabilities and sample spaces, and interpret situations involving sample spaces, probability, and populations.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their

own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
7.8.1		more likely less likely
7.8.2	**event** **chance experiment** **outcome** equally likely as not	likely unlikely impossible certain
7.8.3	**probability** **random** **sample space**	**outcome**
7.8.5	**simulation**	**probability** **random**
7.8.7		**event** **simulation**
7.8.8	tree (diagram)	**sample space**
7.8.9		tree (diagram)
7.8.11	mean absolute deviation (MAD) distribution very different overlap	mean median dot plot
7.8.12	**population** **sample** survey	mean absolute deviation (MAD)
7.8.13	**representative** measure of center	distribution center (of a distribution) spread
7.8.14	random sample	
7.8.15	interquartile range (IQR) measure of variability box plot	**population** **sample** random sample symmetric
7.8.16	**proportion**	**representative**
7.8.17		interquartile range (IQR) measure of variability
7.8.18	meaningful difference	overlap measure of center

lesson	new terminology	
	receptive	productive
7.8.20		meaningful difference

Section A: Probabilities of Single-Step Events

- Lesson 1: Mystery Bags
- Lesson 2: Chance Experiments
- Lesson 3: What Are Probabilities?
- Lesson 4: Estimating Probabilities through Repeated Experiments
- Lesson 5: More Estimating Probabilities
- Lesson 6: Estimating Probabilities Using Simulation

Section B: Probabilities of Multi-step Events

- Lesson 7: Simulating Multi-step Experiments
- Lesson 8: Keeping Track of All Possible Outcomes
- Lesson 9: Multi-step Experiments
- Lesson 10: Designing Simulations

Section C: Sampling

- Lesson 11: Comparing Groups
- Lesson 12: Larger Populations
- Lesson 13: What Makes a Good Sample?
- Lesson 14: Sampling in a Fair Way

Section D: Using Samples

- Lesson 15: Estimating Population Measures of Center
- Lesson 16: Estimating Population Proportions
- Lesson 17: More about Sampling Variability
- Lesson 18: Comparing Populations Using Samples
- Lesson 19: Comparing Populations with Friends

Section E: Let's Put It to Work

- Lesson 20: Memory Test

Unit 9: Putting It All Together

In this optional unit, students use concepts and skills from previous units to solve problems. The first section focuses on calculating or estimating quantities associated with running a restaurant. The second section explores a variety of different contexts, such as population density, Fermi problems, measurement error, and deforestation. In the last section, students build a trundle wheel and design a five-kilometer race course.

All related standards in this unit have been addressed in prior units. These sections provide an optional opportunity for students to go more deeply and make connections between domains.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes, such as justifying, representing, and critiquing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Justify

- Choices and predictions in the context of running a restaurant (Lesson 1).
- Reasoning about length, area, and volume in the context of a restaurant (Lesson 3).
- Reasoning about the forested area on a map (Lesson 8).

Represent

- Costs of ingredients in a spreadsheet (Lesson 1).
- Situations using expressions and equations (Lesson 6).
- A map of a designed race course (Lesson 12).

Critique

- Peer reasoning about calculations of age, heart beats, and hairs (Lesson 5).
- Peer reasoning about percent error in length, area, and volume measurement (Lesson 7).
- Peer methods of measuring distance (Lesson 9).

In addition, students are also expected to describe methods for measuring distance, including how to build and use a trundle wheel, and compare advantages and disadvantages of different methods.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
7.9.1	spreadsheet cell formula serving	
7.9.2	profit expense	
7.9.4	population density	
7.9.7		**percent error**
7.9.8	forested land deforestation reforestation	
7.9.10	trundle wheel	
7.9.12		trundle wheel

Section A: Running a Restaurant

- Lesson 1: Cost of a Meal
- Lesson 2: Costs of Running a Restaurant
- Lesson 3: Restaurant Floor Plan

Section B: Making Connections

- Lesson 4: How Crowded Is This Neighborhood?
- Lesson 5: Fermi Problems
- Lesson 6: More Expressions and Equations
- Lesson 7: Measurement Error
- Lesson 8: Deforestation at Scale

Section C: Designing a Course

- Lesson 9: Measuring Long Distances over Uneven Terrain
- Lesson 10: Building a Trundle Wheel
- Lesson 11: Using a Trundle Wheel to Measure Distances
- Lesson 12: Designing a 5K Course

Pacing Guide

Number of days includes assessments. Upper bound of range includes optional lessons.

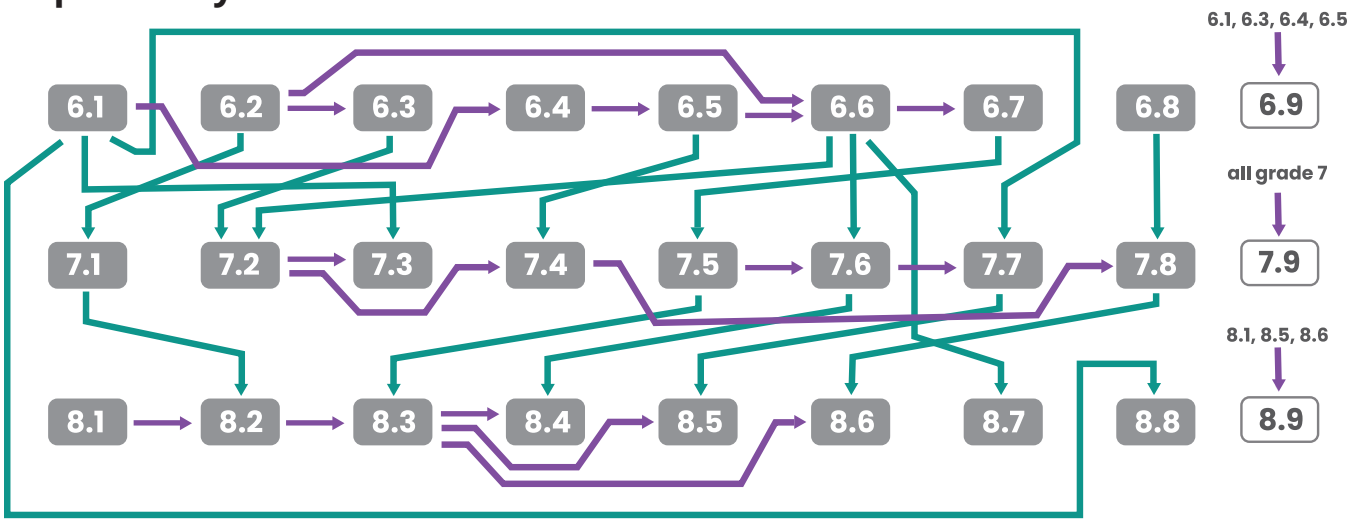
	Grade 6	Grade 7	Grade 8
week 1	Unit 1 Area and Surface Area (20–22 days) (MA) Optional Lesson: 16, 19	Unit 1 Scale Drawings (12–15 days) Optional Lessons: 6, 8, 13	Unit 1 Rigid Transformations and Congruence (20 days) (MA) Optional Lessons: none
week 2			
week 3			
week 4		Unit 2 Introducing Proportional Relationships (16–17 days) Optional Lessons: 14	
week 5	Unit 2 Introducing Ratios (19 days) Optional Lessons: none		Unit 2 Dilations, Similarity, and Introducing Slope (15 days) Optional Lessons: none
week 6			
week 7			
week 8		Unit 3 Measuring Circles (11–13 days) Optional Lessons: 5, 11	Unit 3 Linear Relationships (16–17 days) Optional Lessons: 11
week 9	Unit 3 Unit Rates and Percentages (16–19 days) Optional Lesson: 2, 9, 16		
week 10			
week 11		Unit 4 Proportional Relationships and Percentages (17–18 days) Optional Lesson: 15	
week 12			Unit 4 Linear Equations and Linear Systems (18 days) Optional Lessons: none
week 13	Unit 4 Dividing Fractions (16–20 days) (MA) Optional Lessons: 3, 4, 9, 16		
week 14			
week 15		Unit 5 Rational Number Arithmetic (18–19 days) Optional Lesson: 10	
week 16			
week 17	Unit 5 Arithmetic in Base Ten (15–18 days) (MA) Optional Lessons: 2, 9, 15		Unit 5 Functions and Volume (23–25 days) (MA) Optional Lessons: 18, 22
week 18			
week 19		Unit 6 Expressions, Equations, and Inequalities (25 days) (MA) Optional Lessons: none	
week 20			
week 21	Unit 6 Expressions and Equations (20–22 days) (MA) Optional Lessons: 11, 18		Unit 6 Associations in Data (12–13 days) Optional Lesson: 11
week 22			
week 23			
week 24		Unit 7 Angles, Triangles, and Prisms (18–19 days) Optional Lesson: 17	Unit 7 Exponents and Scientific Notation (18 days) Optional Lessons: none
week 25	Unit 7 Rational Numbers (20–21 days) Optional Lesson: 19		
week 26			
week 27			
week 28		Unit 8 Probability and Sampling (21–23 days) (MA) Optional Lessons: 17, 20	Unit 8 Pythagorean Theorem and Irrational Numbers (20 days) Optional Lessons: none
week 29	Unit 8 Data Sets and Distributions (20–21 days) (MA) Optional Lessons: 18		
week 30			
week 31			
week 32		Unit 9 Putting It All Together (0–12 days) Optional Lessons: all	Unit 9 Putting It All Together (0–6 days) Optional Lessons: all
week 33	Unit 9 Putting It All Together (0–11 days) Optional Lessons: all		
week 34			
week 35			

(MA) = Unit has Mid-Unit Assessment

Total number of days for each course = Lessons + Assessments – Optional Lessons

Grade 6 = 146 Days Grade 7= 138 Days Grade 8 = 142 Days

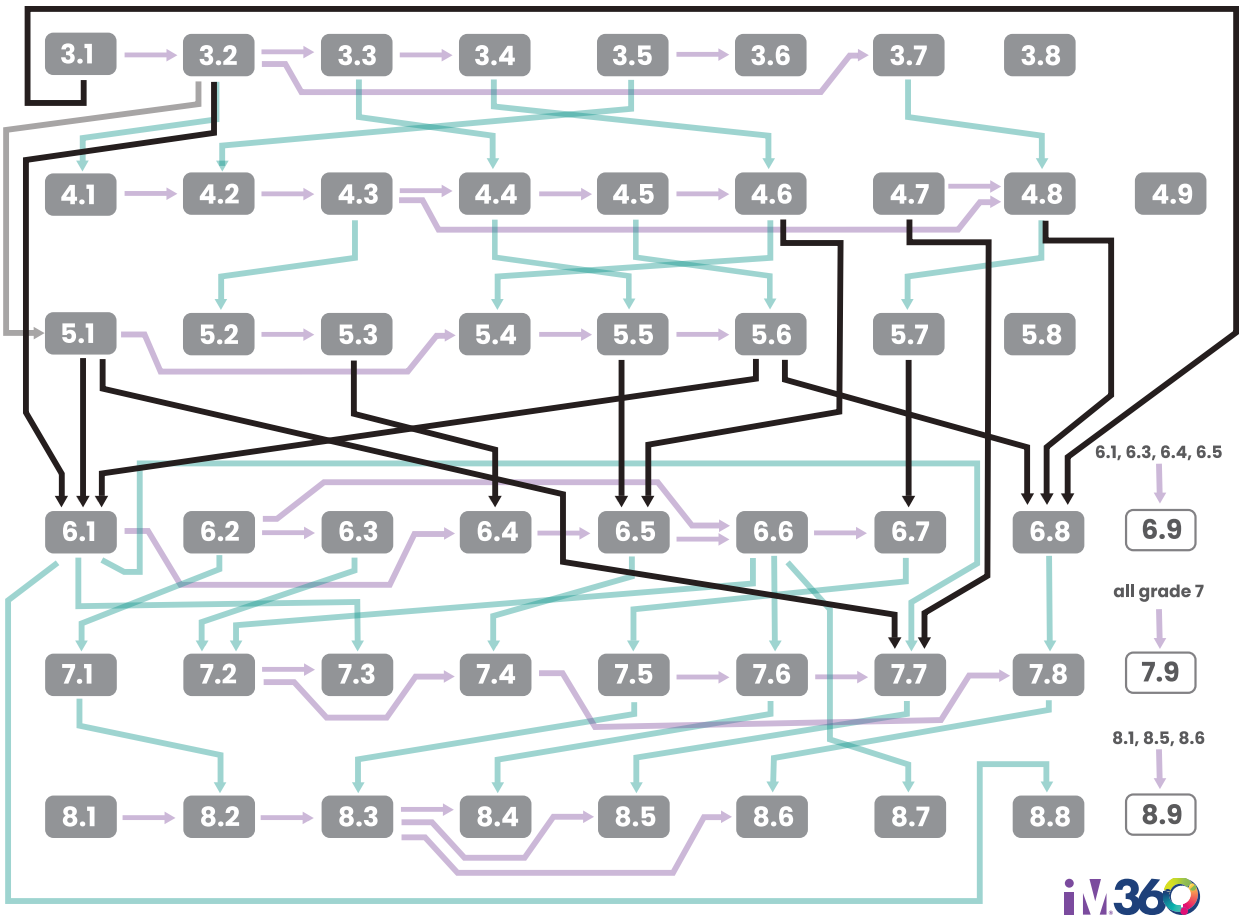
Dependency Chart



In the unit dependency chart, an arrow indicates that a particular unit is designed for students who already know the material in a previous unit. Reversing the order of the units would have a negative effect on mathematical or pedagogical coherence. Examples:

- There is an arrow from 6.2 to 6.6. Students are expected to use their knowledge of contexts involving ratios (from 6.2) to write and solve equations representing such contexts (in 6.6).
- There is an arrow from 7.4 to 7.8. Students are expected to use their skills in representing percentages (from 7.4) when solving problems about probability (in 7.8).
- There is an arrow from 8.3 to 8.6. Students are expected to use their skills in writing and interpreting an equation that represents a line (from 8.3) to interpret the parameters in an equation that represents a line that fits a scatter plot (in 8.6).

The following chart shows unit dependencies across the curriculum for IM Grades 3–8.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all grade 7
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8.1, 8.5, 8.6
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