Understanding Rational Inputs

Let's look at exponential functions where the input values are not whole numbers.

3.1 Keeping Equations True

1. Select **all** solutions to $x \cdot x = 5$. Be prepared to explain your reasoning.

   A. $\frac{1}{25}$

   B. $\sqrt{5}$

   C. $\frac{5}{2}$

   D. $5^{\frac{1}{2}}$

   E. $\frac{\sqrt{5}}{2}$

   F. $\sqrt{25}$

2. Select **all** solutions to $p \cdot p \cdot p = 10$. Be prepared to explain your reasoning.

   A. $10^{\frac{1}{3}}$

   B. $\sqrt{10}$

   C. $\frac{10}{3}$

   D. $\frac{\sqrt{10}}{3}$

   E. $\sqrt[3]{10}$

   F. $\frac{1}{3}\sqrt{10}$

3.2 Moon Base Population

In a video game, Jada is building a moon base to support a growing population and to deal with challenges. Jada's base has a population of 54,500 in the year 2240, and between 2240 and 2270 the population of the base grows exponentially by about 60% per decade.

1. Find the population of Jada's moon base in 2250 and in 2260 according to this model.

2. The population is a function f of the number of decades d after 2240. Write an equation for f.

3. a. Explain what $f(0.5)$ means in this situation.

   b. Graph your function using graphing technology, and estimate the value of $f(0.5)$.

   c. Explain why we can find the value of $f(0.5)$ by multiplying 54,500 by $\sqrt{1.6}$. Find that value.

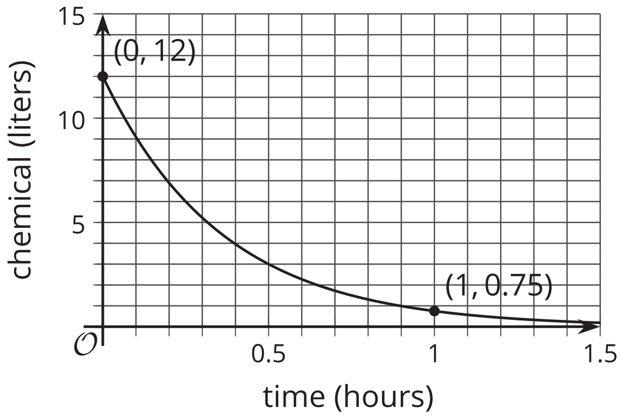
4. Based on the model, what was the population of Jada's base in 2258? Show your reasoning.

Are you ready for more?

Andre said, "The population of Jada's moon base increased by the same percentage between 2242 and 2252 and between 2247 and 2257." Do you agree with his statement? Explain or show your reasoning.

3.3 Cleaning Up a Spill

A chemical is accidentally spilled into a lake and needs to be cleaned up. The cleaning process decreases the amount of the chemical in the lake roughly exponentially. Here is a graph representing f, an exponential function that models the amount of chemical left in the lake, t hours after the cleaning begins.



1. Use the graph to estimate $f\left(\frac{1}{3}\right)$, and explain what it tells us in this situation.

2. After one hour, 0.75 liters of the chemical remains in the lake. Find an equation that defines f.

Are you ready for more?

By what percentage does the amount of chemical in the lake decay every 10 minutes? Explain or show your reasoning.

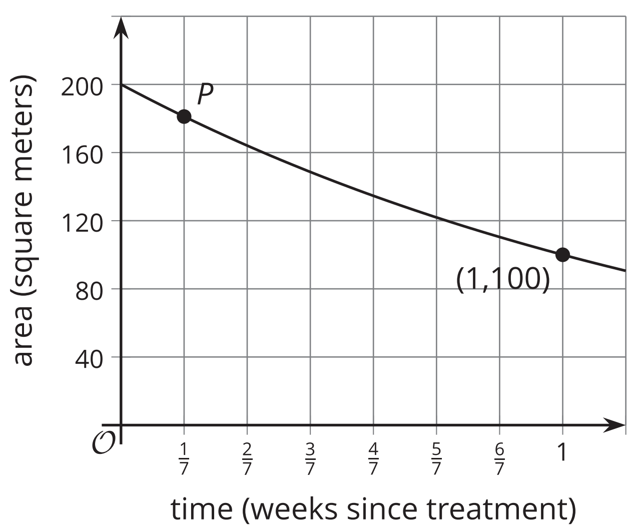
Lesson 3 Summary

Some exponential functions can have inputs that are any numbers on the number line, not just integers.

Suppose the area of a pond covered by algae A, in square meters, is modeled by $A = 200 \cdot \left(\frac{1}{2}\right)^w$, where w is the number of weeks since a treatment was applied to the pond. How could we use this equation to determine the area covered after 1 day?

Well, since $w = 1$ is one week and each week has 7 days, $w = \frac{1}{7}$ is 1 day. So after 1 day, the algae covers $200\left(\frac{1}{2}\right)^{\frac{1}{7}}$ square meters, or about 181 square meters. Using a calculator, we know that the expression $\left(\frac{1}{2}\right)^{\frac{1}{7}}$, which is equivalent to $\sqrt[7]{\frac{1}{2}}$, is about 0.906. This means that after 1 day, only 91% of the algae from the previous day remains.

This information can also be seen on a graph representing the area. The point at $(1, 100)$ marks the area covered by the algae after 1 week. Point P marks the covered area after $\frac{1}{7}$ of a week, or one day.



The graph can be used to estimate the vertical coordinate of P and shows that it is close to 180.