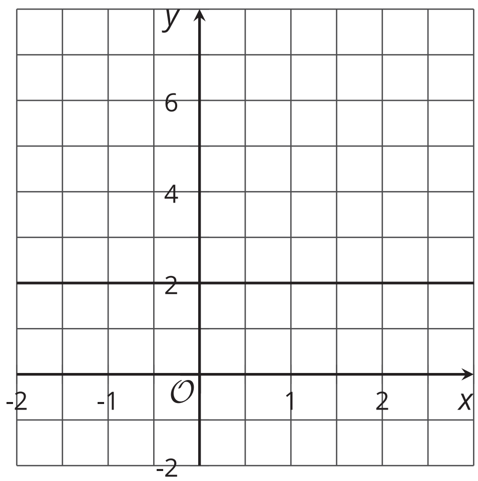
Building Quadratic Functions to Describe Situations (Part 3)

Let's look at how to maximize revenue.

7.1 Which Three Go Together: Graphs of Four Functions

Which three go together? Why do they go together?

A



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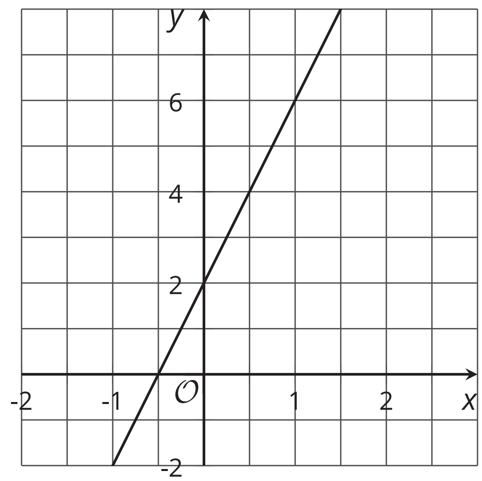

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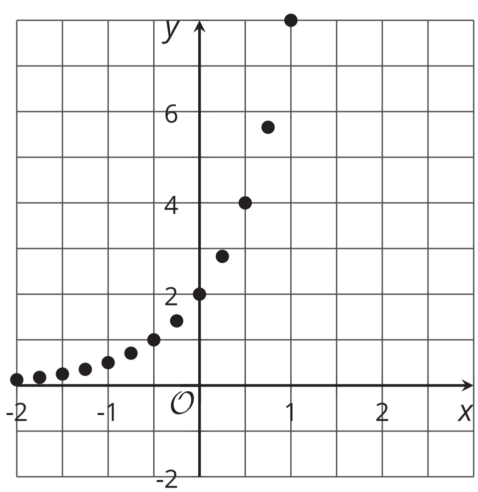

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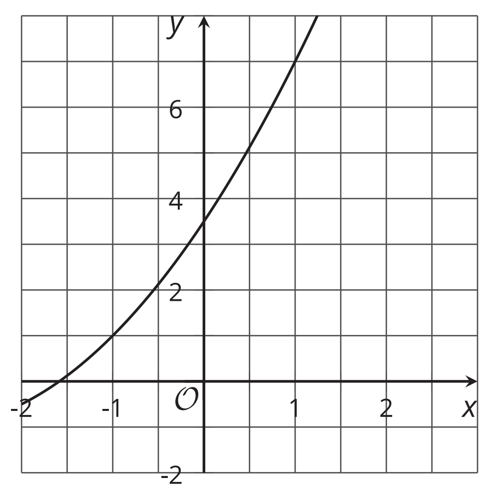

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7.2 What Price to Charge?

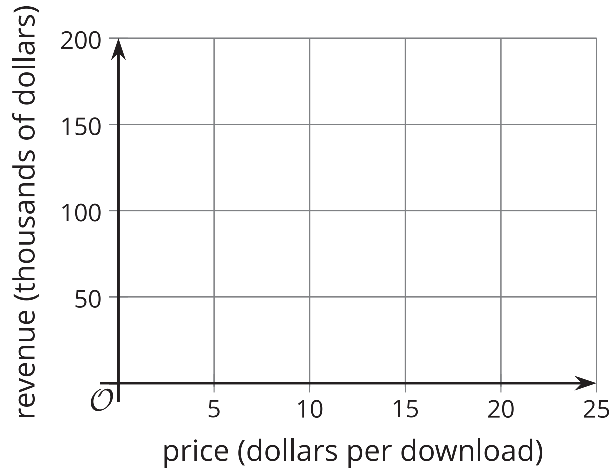
A company that sells movies online is deciding how much to charge customers to download a new movie. Based on data from previous sales, the company predicts that if they charge x dollars for each download, then the number of downloads, in thousands, is $18 - x$.

1. Complete the table to show the predicted number of downloads at each listed price. Then find the revenue at each price. The first row has been completed for you.

price (dollars per download)	number of downloads (thousands)	revenue (thousands of dollars)
3	15	45
5		
10		
12		
15		
18		
x		

2. Is the relationship between the price of the movie and the revenue (in thousands of dollars) quadratic? Explain how you know.

3. Plot the points that represent the revenue, r, as a function of the price of one download in dollars, x.



4. What price would you recommend that the company charge for a new movie? Explain your reasoning.

Are you ready for more?

The function that uses the price (in dollars per download), x, to determine the number of downloads (in thousands), $18 - x$, is an example of a demand function and its graph is known. Economists are interested in factors that can affect the demand function, and therefore the price, that suppliers wish to set.

1. What are some things that could increase the number of downloads predicted for the same given prices?

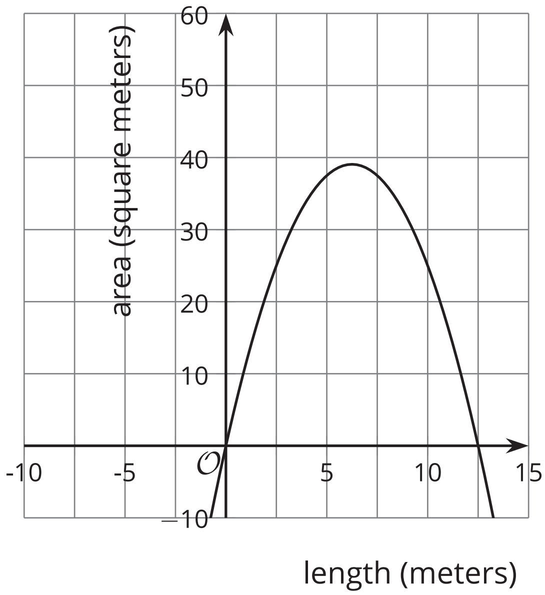
2. If the demand shifted so that we predicted $20 - x$ thousand downloads at a price of x dollars per download, what do you think will happen to the price that gives the maximum revenue? Check what actually happens.

7.3 Domain, Vertex, and Zeros

Here are four sets of descriptions and equations that represent some familiar quadratic functions. The graphs show what graphing technology may produce when the equations are graphed. For each function:

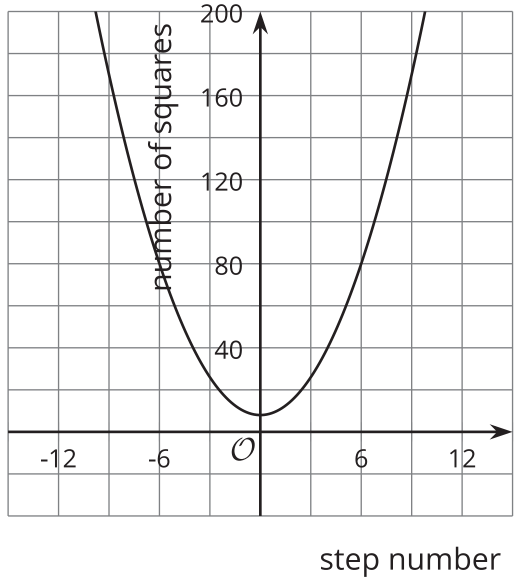
- Describe a domain that is appropriate for the situation. Think about any upper or lower limits for the input, as well as whether all numbers make sense as the input. Then, describe how the graph should be modified to show the domain that makes sense.
- Identify or estimate the vertex on the graph. Describe what it means in the situation.
- Identify or estimate the zeros of the function. Describe what they meant in the situation.

1. The area of a rectangle with a perimeter of 25 meters and a side length of x: $A(x) = x \cdot \frac{(25-2x)}{2}$



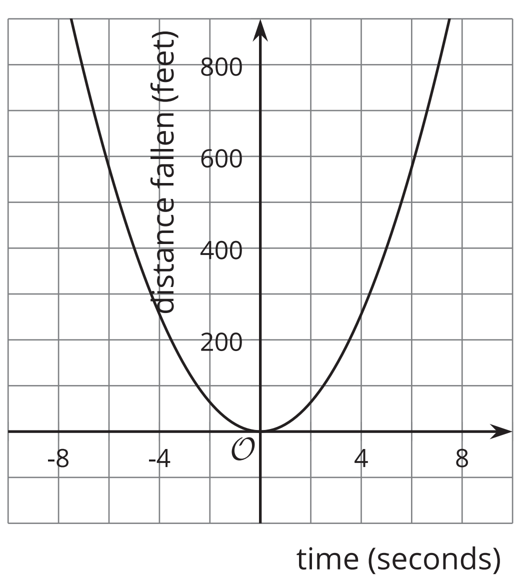
- Domain:
- Vertex:
- Zeros:

2. The number of squares as a function of step number n: $f(n) = n^2 + 4$



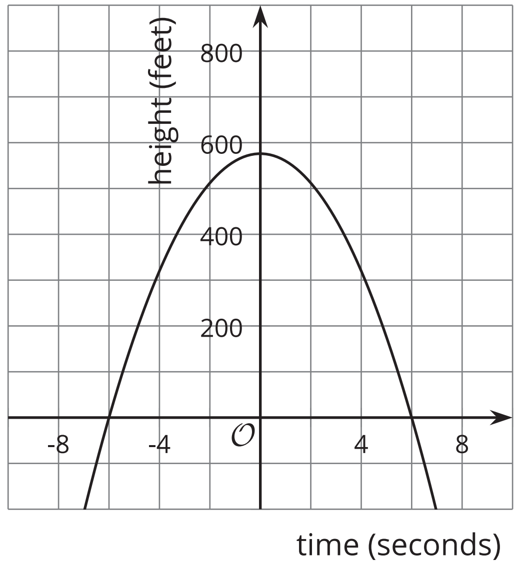
- Domain:
- Vertex:
- Zeros:

3. The distance, in feet, that an object has fallen t seconds after being dropped: $g(t) = 16t^2$



- Domain:
- Vertex:
- Zeros:

4. The height, in feet, of an object t seconds after being dropped: $h(t) = 576 - 16t^2$



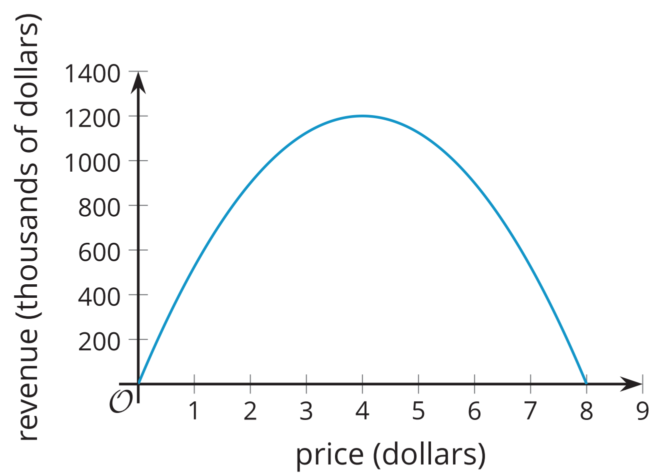
- Domain:
- Vertex:
- Zeros:

Lesson 7 Summary

Quadratic functions often come up when studying revenue, which is the amount of money collected when selling something.

Suppose we are selling raffle tickets and deciding how much to charge for each ticket. When the price of the tickets is higher, typically fewer tickets will be sold.

Let's say that with a price of d dollars, it is possible to sell $600 - 75d$ tickets. We can find the revenue by multiplying the price by the number of tickets expected to be sold. A function that models the revenue, r, collected is $r(d) = d(600 - 75d)$. Here is a graph that represents the function.



When ticket prices are low, a lot of tickets may be sold, but the total revenue is still low because the tickets are cheap. When the ticket prices get close to \$8, not many tickets are sold so the revenue is low again. From the graph, we can tell that the greatest revenue comes when there is a balance between ticket price and number of tickets sold. In this situation, that is \$1,200 of revenue when tickets are sold for \$4 each.

We can also see that for function r, the domain is between 0 and 8. This makes sense because the cost of the tickets can't be negative. If the price is more than \$8, the model doesn't work because the revenue collected can't be negative. A negative revenue (based on a non-negative ticket price) could occur only if the number of tickets sold is negative, which is not possible.

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