Unit 7, Lesson 2

Multiplying Powers of 10

Goals

- Generalize a process for finding a power raised to a power, and justify (orally and in writing) that $(10^n)^m = 10^{n \cdot m}$.
- Generalize a process for multiplying exponential expressions with the same base, and justify (orally and in writing) that $10^n \cdot 10^m = 10^{n+m}$.

Learning Targets

- I can explain and use a rule for multiplying powers of 10.
- I can explain and use a rule for raising a power of 10 to a power.

Lesson Narrative

In this lesson, students make use of repeated reasoning to discover the exponent rules $10^n \cdot 10^m = 10^{n+m}$ and $(10^n)^m = 10^{n \cdot m}$.

Students begin by relating base-ten diagrams to exponents, specifically powers of 10. This allows students to consider the meaning of quantities and not just how to compute them (MP2).

Next, students expand expressions written as the product of two powers of 10 or written as a value with an exponent raised to another power, and notice patterns when asked to write the expression using a single power of 10 (MP8). Students will extend this exponent rule to cases where the exponents are zero or negative in following lessons, but the focus here is on cases with positive exponents.

Standards

Addressing	**8.EE.A.1**
Building Toward	**8.EE.A.1, 8.EE.A.3, 8.EE.A.4**

Instructional Routines

- MLR2: Collect and Display

Required Materials

Materials to Gather

- Math Community Chart: Cool-down

Required Preparation

Activity 2:

Create a visual display of the exponent rule $10^n \cdot 10^m = 10^{n+m}$ to be displayed for all to see throughout the unit. A sample display can be seen in the *Activity Synthesis*.

Activity 3:

Create a visual display (or add to an existing display) of the exponent rule $(10^n)^m = 10^{n \cdot m}$ to be displayed for all to see throughout the unit. A sample display can be seen in the *Activity Synthesis*.

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Student Facing Learning Goals

Let's explore patterns with exponents when we multiply powers of 10.

2.1 Picture a Power of 10

Warm-up

5 min

Activity Narrative

The purpose of this activity is for students to develop a sense of visual scale between powers of 10. Students should understand that multiplying by 10 corresponds to increasing the exponent by 1.

This activity prompts students to make sense of quantities and their relationships (MP2). For example, even though the notation for 10^{100} does not appear to be much different than 10^{98}, it is 100 times larger. Small changes in the exponent can result in large changes in the value of the expression.

Standards

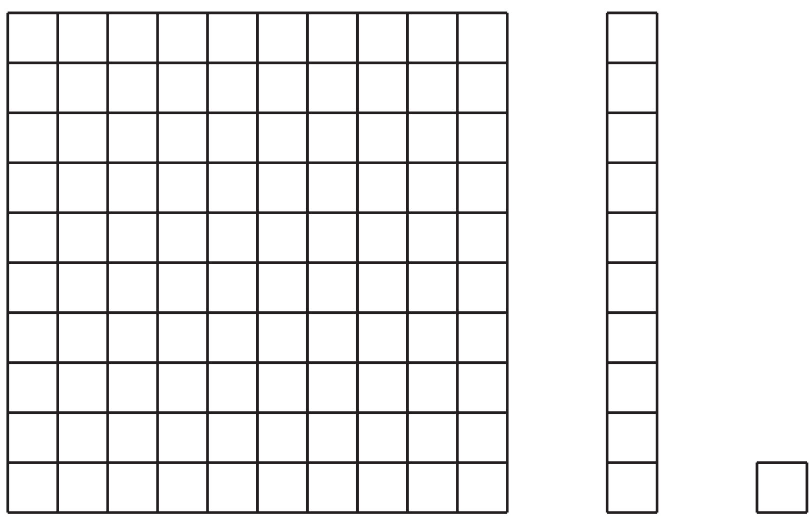
Building Toward **8.EE.A.1, 8.EE.A.3, 8.EE.A.4**

Launch

Arrange students in groups of 2. Give students 1–2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

In the diagram, the medium rectangle is made up of 10 small squares. The large square is made up of 10 medium rectangles.



1. How could the large square be represented as a power of 10? Explain your reasoning.
2. If each small square represents 10^2, then what does the medium rectangle represent? The large square?
3. If each small square represents 10^5, then what does the medium rectangle represent? The large square?

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Student Response

1. The large square can be represented by 10^2 because there are 100 small squares.
2. The medium rectangle rectangle represents 10^3 because $10^2 \cdot 10 = 10^3$. The large square represents 10^4 because $10^2 \cdot 10^2 = 10^4$.
3. The medium rectangle rectangle represents 10^6 because $10^5 \cdot 10 = 10^6$. The large square represents 10^7 because $10^5 \cdot 10^2 = 10^7$.

Activity Synthesis

The purpose of this discussion is to make sure students understand that increasing the exponent of a power of 10 by 1 corresponds to multiplying by 10, and decreasing the exponent by 1 corresponds to dividing by 10.

Ask students the following questions:

- "If the medium rectangle represents 10^4, then what does the large square represent? The small square? (The large square represents 10^5 because $10^4 \cdot 10 = 10^5$. The small square represents 10^3 because $10^4 \div 10 = 10^3$.)
- "If the large square represents 10^{100}, then what does the medium rectangle represent? The small square?" (The medium rectangle rectangle represents 10^{99} because $10^{100} \div 10 = 10^{99}$ and the small square represents 10^{98} because $10^{100} \div 10^2 = 10^{98}$.)

2.2 Multiplying Powers of Ten

15 min

Activity Narrative

The goal of this activity is to help students flexibly transition between different notations for powers of 10 and to introduce the property of multiplication of values with the same base. Students observe patterns in the notations and generalize that $10^n \cdot 10^m = 10^{n+m}$ for the values of n and m that are positive integers (MP8).

Standards

Addressing **8.EE.A.1**

Launch

Give students 1 minute of quiet think time to complete the first unfinished row in the table before inviting 1–2 students to share and explain their answers. When it is clear that students understand how to complete the table, explain that they can skip one entry in the table, but they have to be able to explain why they skipped it. Give students 5–6 minutes to complete the remaining questions before a whole-class discussion.

Student Task Statement

1. a. Complete the table to explore patterns in the exponents when multiplying powers of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

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expression	expanded	single power of 10
$10^2 \cdot 10^3$	$(10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^5
$10^4 \cdot 10^3$		
$10^4 \cdot 10^4$		
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$	
$10^{18} \cdot 10^{23}$		

b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. a. Use the patterns you found in the table to rewrite $10^n \cdot 10^m$ as an equivalent expression with a single exponent, like $10^{\square}$.

   b. Use your rule to write $10^4 \cdot 10^0$ with a single exponent. What does this tell you about the value of 10^0?

3. The state of Georgia has roughly 10^7 human residents. Each human has roughly 10^{13} bacteria cells in his or her digestive tract. How many bacteria cells are there in the digestive tracts of all the humans in Georgia?

Student Response

1. a.

expression	expanded	single power of 10
$10^2 \cdot 10^3$	$(10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^5
$10^4 \cdot 10^3$	$(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^7
$10^4 \cdot 10^4$	$(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10)$	10^8
$10^3 \cdot 10^5$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$	10^8
$10^{18} \cdot 10^{23}$	skip	10^{41}

   b. I chose to skip the expanded column of $10^{18} \cdot 10^{23}$ because there are too many factors that are 10 and they won't fit in the table.

2. a. $10^n \cdot 10^m = 10^{n+m}$ because multiplying n factors that are 10 with m factors that are 10 results in $n + m$ factors that are 10.

   b. 10^4 because $10^4 \cdot 10^0 = 10^{4+0}$. That means 10^0 must equal 1 for the rule to work.

3. There are 10^{20} bacteria because 10^7 people times 10^{13} bacteria per person is equal to 10^{20} total bacteria.

Are You Ready for More?

There are four ways to make 10^4 by multiplying powers of 10 with smaller, positive exponents.

$$10^1 \cdot 10^1 \cdot 10^1 \cdot 10^1$$

$$10^1 \cdot 10^1 \cdot 10^2$$

$$10^1 \cdot 10^3$$

$$10^2 \cdot 10^2$$

(This list is complete if you don't pay attention to the order you write them in. For example, we are only counting $10^1 \cdot 10^3$ and $10^3 \cdot 10^1$ once.)

1. How many ways are there to make 10^6 by multiplying smaller powers of 10 together?
2. How about 10^7? 10^8?

Extension Student Response

1. 10 ways
2. 14 ways, 22 ways

Activity Synthesis

The goal of this discussion is to reinforce the exponent rule for multiplying exponential expressions with the same base. Introduce and explain the visual display prepared earlier. This display should be kept visible to students throughout the remainder of the unit.

Rule	Example showing how it works
$10^n \cdot 10^m = 10^{n+m}$	$10^2 \cdot 10^3 = (10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) = 10^5$ two factors that are ten $\cdot$ three factors that are ten $=$ five factors that are ten

Continue to reinforce student understanding of this rule by writing out the expanded form of each expression and counting the total number of factors of 10 when discussing the following questions:

- "What is $10^3 \cdot 10^1$ written as a single power of 10?" ($10^3 \cdot 10^1 = (10 \cdot 10 \cdot 10) \cdot (10) = 10^4$)
- "What is $10^5 \cdot 10^2$ written as a single power of 10?" ($10^5 \cdot 10^2 = (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10) = 10^7$)
- "What is $10^{20} \cdot 10^{17}$ written as a single power of 10?" ($10^{20} \cdot 10^{17} = 10^{20+17} = 10^{37}$)
- "What are some different ways to write 10^8 as the product of 2 powers of 10?" ($10^4 \cdot 10^4$ and $10^1 \cdot 10^7$)

2.3 Raising Powers of 10 to Another Power

15 min

Activity Narrative

In this activity, students explore patterns to discover the property $(10^n)^m = 10^{n \cdot m}$ for values of n and m that are positive integers (MP8).

Standards

Addressing **8.EE.A.1**

Instructional Routines

- MLR2: Collect and Display

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Launch

Give students 1 minute of quiet think time to complete the first unfinished row in the table before inviting 1–2 students to share and explain their answers. When it is clear that students understand how to complete the table, explain to them that they can skip one entry in the table, but they have to be able to explain why they skipped it. Give students 5–6 minutes to complete the remaining questions before a whole-class discussion.

Access for English Language Learners

MLR2 Collect and Display. Collect the language that students use to expand expressions with a power raised to another power. Display words and phrases such as "factors," and "groups of factors," "multiply the exponents." During the *Activity Synthesis,* invite students to suggest ways to update the display: "What are some other words or phrases we should include?" Invite students to borrow language from the display as needed.
Advances: Conversing, Reading

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Have students complete one problem at a time. Check in with students to provide feedback and encouragement after each chunk.
Supports accessibility for: Attention, Social-Emotional Functioning

Student Task Statement

1. a. Complete the table to explore patterns in the exponents when raising a power of 10 to a power. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$(10^3)^2$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^6
$(10^2)^5$	$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$	
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	
$(10^4)^2$		
$(10^8)^{11}$		

   b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. Use the patterns you found in the table to rewrite $(10^n)^m$ as an equivalent expression with a single exponent, like $10^{\square}$.

3. If you took the amount of oil consumed in 2 months in 2013 worldwide, you could make a cube of oil that measures 10^3 meters on each side. How many cubic meters of oil is this? Do you think this would be enough to fill a pond, a lake, or an ocean?

Student Response

1. a.

expression	expanded	single power of 10
$(10^3)^2$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^6
$(10^2)^5$	$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$	10^{10}
$(10^3)^4$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^{12}
$(10^4)^2$	$(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10)$	10^8
$(10^8)^{11}$	skip	10^{88}

   b. I chose to skip the expanded column of $(10^8)^6$ because the table cannot fit 88 factors that are 10.

2. $(10^n)^m = 10^{n \cdot m}$ because there are m groups of n factors that are 10.
3. 10^9 cubic meters of oil because $(10^3)^3 = 10^9$. Sample response: This is enough to fill a lake.

Are You Ready for More?

$2^{12} = 4{,}096$. How many other whole numbers can you raise to a power and get 4,096? Explain or show your reasoning.

Extension Student Response

Since $4{,}096 = 2^{12}$, it can be broken down into other representations of the form $(2^m)^n$ so that $m \cdot n = 12$. For example, $(2^2)^6 = 4^6$, $(2^3)^4 = 8^4$, $(2^4)^3 = 16^3$, $(2^6)^2 = 64^2$, and $(2^{12})^1 = 4{,}096^1$.

Activity Synthesis

The goal of this discussion is to reinforce the exponent rule for raising powers of 10 to another power. Introduce and explain the visual display prepared earlier. This display should be kept visible to students throughout the remainder of the unit.

Rule	Example showing how it works
$(10^n)^m = 10^{n \cdot m}$	$(10^2)^3 = (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) = 10^6$
	three groups of two factors that are ten = six factors that are ten

Continue to reinforce student understanding of this rule by writing out an expanded form of each expression when discussing the following questions:

- "What is $(10^2)^4$ written as a single power of 10?" ($(10^2)^4 = (10^2) \cdot (10^2) \cdot (10^2) \cdot (10^2) = 10^{2 \cdot 4} = 10^8$)
- "What is $(10^7)^3$ written as a single power of 10?" ($(10^7)^3 = (10^7) \cdot (10^7) \cdot (10^7) = 10^{7 \cdot 3} = 10^{21}$)
- "What are some different ways to write 10^{24} as a power of 10 raised to another power?" ($(10^4)^6$, $(10^{12})^2$, $(10^3)^8$)

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Lesson Synthesis

The purpose of this discussion is to emphasize the similarities and differences between the rule for multiplying values with the same base and the rule for raising a value with an exponent to another power. Display these two statements for all to see, and explain that Andre and Elena were both trying to write $10^2 \cdot 10^2 \cdot 10^2$ with a single exponent.An

Andre: $10^2 \cdot 10^2 \cdot 10^2 = 10^{2+2+2} = 10^6$

Elena: $10^2 \cdot 10^2 \cdot 10^2 = (10^2)^3 = 10^{2+3} = 10^5$

Ask students if they agree with either Andre or Elena. Give students time to share their reasoning with a partner before inviting students to share their thinking with the class. Record student responses for all to see. Consider asking the following questions for discussion:

- "How are these statements the same? How are they different?" (Both statements are attempting to write $10^2 \cdot 10^2 \cdot 10^2$ as a number with a single exponent. The first statement uses the rule for multiplying values with the same base while the second statement tries to use the rule for raising a value with an exponent to another power.)
- "How can the second statement be corrected to make it true?" (When raising a value with an exponent to another power, the two exponents can be multiplied together, giving $(10^2)^3 = 10^{(2 \cdot 3)} = 10^6$.)
- "How can $10^5 \cdot 10^5 \cdot 10^5 \cdot 10^5$ be written with exponents instead of repeated multiplication?" ($(10^5)^4 = 10^{20}$)

2.4 Making a Million

Cool-down

10 min

Teacher Notes for IM 6–8 Math Accelerated v.360
Decrease the timing of this activity to 5 minutes.

Standards

Addressing **8.EE.A.1**

Launch

Math Community
Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

- "What norm(s) should stay the way they are?"
- "What norm(s) do you think should be made more clear? How?"
- "What norms are missing that you would add?"
- "What norm(s) should be removed?"

Ask students to respond to one or more of the questions after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made, to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about. Discuss the potential revisions over the next few lessons.

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Student Task Statement

Here are some equivalent ways of writing 10^4:

- 10,000
- $10 \cdot 10^3$
- $(10^2)^2$

Write as many expressions as you can that have the same value as 10^6.

Student Response

Answers vary. Sample responses:

- 1,000,000
- $1,000 \cdot 1,000$
- $10^2 \cdot 10^4$
- $(10^3)^2$
- $1 \cdot 10^6$
- $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

Responding to Student Thinking

Points to Emphasize
If most students struggle with multiplying powers of 10, make time to do the optional activity refered to here. Focus on equivalent expressions using multiplication.

Accelerated 7, Unit 7, Lesson 3, Activity 4 Making Millions

Lesson 2 Summary

In this lesson, we developed a rule for multiplying powers of 10: Multiplying powers of 10 corresponds to adding the exponents together.

Rule	Example showing how it works
$10^n \cdot 10^m = 10^{n+m}$	$10^2 \cdot 10^3 = (10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) = 10^5$
	two factors that are ten $\cdot$ three factors that are ten = five factors that are ten

To see this, multiply 10^2 and 10^3. We know that 10^2 has two factors that are 10 and that 10^3 has three factors that are 10. That means that $10^2 \cdot 10^3$ has 5 factors that are 10.

This will work for other powers of 10, too. For example, $10^{14} \cdot 10^{47} = 10^{(14+47)} = 10^{61}$.

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We also developed a rule for raising a power of 10 to another power: Taking a power of 10 and raising it to another power is the same as multiplying the exponents.

Rule	Example showing how it works
$(10^n)^m = 10^{n \cdot m}$	$(10^2)^3 = (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) = 10^6$ three groups of two factors that are ten = six factors that are ten

To understand this, take 10^2 and raise it to the power of 3. We know that 10^2 has two factors that are 10. Raising 10^2 to the power of 3 means that there are three groups of two factors that are 10, for a total of 6 factors that are 10, or 10^6.

This works for any power of 10 raised to another power. For example, $(10^6)^{11} = 10^{(6 \cdot 11)} = 10^{66}$.

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Lesson 2 Practice Problems

1 Student Task Statement

Write each expression as a single power of 10:

a. $10^3 \cdot 10^9$

b. $10 \cdot 10^4$

c. $10^{10} \cdot 10^7$

d. $10^3 \cdot 10^3$

e. $10^5 \cdot 10^{12}$

f. $10^6 \cdot 10^6 \cdot 10^6$

Solution

a. 10^{12}

b. 10^5

c. 10^{17}

d. 10^6

e. 10^{17}

f. 10^{18}

2 Student Task Statement

A large rectangular swimming pool is 1,000 feet long, 100 feet wide, and 10 feet deep. The pool is filled to the top with water. Express your answers both as a single power of 10 and as a number in standard form.

a. What is the area of the surface of the water in the pool?

b. How much water does the pool hold?

Solution

a. 10^5 square feet, 100,000 square feet

b. 10^6 cubic feet, 1,000,000 cubic feet

3 Student Task Statement

Write each expression with a single exponent:

a. $(10^7)^2$

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b. $(10^9)^3$

c. $(10^6)^3$

d. $(10^2)^3$

e. $(10^3)^2$

f. $(10^5)^7$

Solution

a. 10^{14}

b. 10^{27}

c. 10^{18}

d. 10^6

e. 10^6

f. 10^{35}

4 Student Task Statement

You have 1,000,000 number cubes, each measuring 1 inch on a side.

a. If you stacked the cubes on top of one another to make an enormous tower, how high would they reach? Explain your reasoning.

b. If you arranged the cubes on the floor to make a square, would the square fit in your classroom? What would its dimensions be? Explain your reasoning.

c. If you layered the cubes to make one big cube, what would be the dimensions of the big cube? Explain your reasoning.

Solution

a. 1,000,000 inches tall, or about 83,000 feet, or about 16 miles. Sample reasoning: $1{,}000{,}000 \div 12 \approx 83{,}333$ feet, which is almost 16 miles ($83{,}333 \div 5{,}280 \approx 15.78$).

b. 1,000-inch side length. Sample reasoning: $1{,}000 \cdot 1{,}000 = 1{,}000{,}000$. 1,000 inches is about $83\frac{1}{3}$ feet long and would probably be longer than most classrooms.

c. 100-inch side length. Sample reasoning: $100 \cdot 100 \cdot 100 = 1{,}000{,}000$. This is a side length of $8\frac{1}{3}$ feet.

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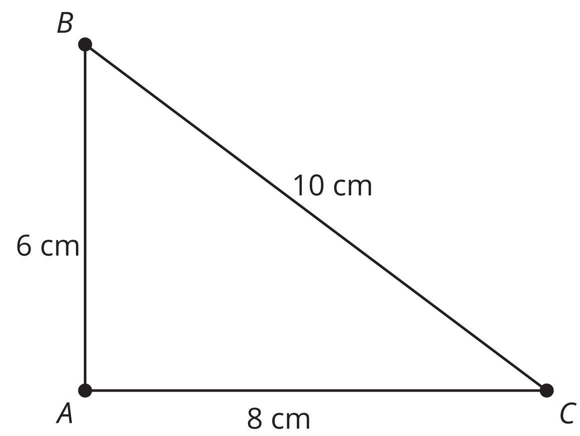
5 from Unit 2, Lesson 12

Student Task Statement

Here is triangle ABC. Triangle DEF is similar to triangle ABC, and the length of EF is 5 cm. What are the lengths of sides DE and DF, in centimeters?



Solution

$DE = 3$ and $DF = 4$, because the scale factor is $\frac{1}{2}$, so each side length is half the corresponding side length of ABC

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