# Expressing Transformations of Functions Algebraically

Let's express transformed functions algebraically.

## 7.1 Describing Translations

Let $g(x) = \sqrt{x}$. Complete the table. Be prepared to explain your reasoning.

| words (the graph of $y = g(x)$ is . . .) | function notation | expression |
|---|---|---|
| translated left 5 units | $g(x + 5)$ | |
| translated left 5 units and down 3 units | | $\sqrt{x + 5} - 3$ |
| | $g(\text{-}x)$ | $\sqrt{\text{-}x}$ |
| translated left 5 units, then down 3 units, then reflected across the $y$-axis | | |

# 7.2 Info Gap: Transforming Functions

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____because _____."
4. Continue to ask questions until you have enough information to solve the problem.
5. Once you have enough information, share the problem card with your partner, and solve the problem independently.
6. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!
4. These steps may be repeated.
5. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
6. Share the data card, and discuss your reasoning.


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# 7.3 Translating Vertex Form

Let $f$ be the function given by $f(x) = x^2$.

1. Write an equation for the function $g$ whose graph is the graph of $f$ translated 3 units left and up 5 units.

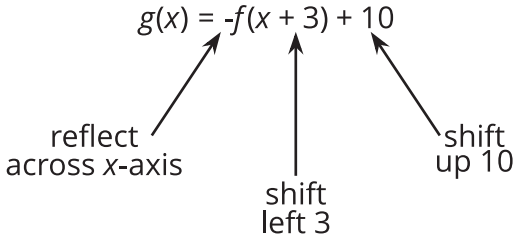
2. What is the vertex of the graph of $g$? Explain how you know.

3. Write an equation for a quadratic function $h$ whose graph has a vertex at $(1.5, 2.6)$.

4. Write an equation for a quadratic function $k$ whose graph opens downward and has a vertex at $(3.2, -4.7)$.

## Lesson 7 Summary

You can use the equation of a function to write an equation for its transformation. For example, let $f(x) = x^2$. Take the graph of $f$, reflect it across the $x$-axis, translate it up 10 units, and translate it left 3 units. What is an equation for this new function? The new function $g$ is related to $f$ by $g(x) = -f(x + 3) + 10$, since




Which means $g(x) = -(x + 3)^2 + 10$.

Sometimes you can recognize from the expression for a function that it is the transformation of a simpler function. For example, consider:

$$H(t) = 10 - (1.2)^{t+5}$$

One way to obtain the expression for $H$ from $1.2^t$ is:

- adding 5 to the input to get $(1.2)^{t+5}$
- multiplying the output by -1 to get $-(1.2)^{t+5}$
- adding 10 to the output to get $10 - (1.2)^{t+5}$

So the graph of $H$ is obtained from the graph of $f(t) = 1.2^t$ by translating left 5 units, reflecting across the $x$-axis, and translating up 10 units. Consider the point $(0, 1)$ on the graph of $f$. After translating, reflecting, and translating again, it becomes the point $(-5, 9)$ on the graph of $H$.