Writing Equations to Model Relationships (Part 2)

Let's use patterns to help us write equations.

3.1 Finding a Relationship

Here is a table of values. The two quantities, x and y, are related.

x	y
1	0
3	8
5	24
7	48

What are some strategies you could use to find a relationship between x and y? Brainstorm as many ways as possible.

3.2 Something about 400

1. Describe in words how the two quantities in each table are related.

   - Table A

number of laps, x	0	1	2.5	6	9
meters run, y	0	400	1,000	2,400	3,600

   - Table B

meters from home, x	0	75	128	319	396
meters from school, y	400	325	272	81	4

   - Table C

electricity bills in dollars, x	85	124	309	816
total expenses in dollars, y	485	524	709	1,216

   - Table D

monthly salary in dollars, x	872	998	1,015	2,110
amount deposited in dollars, y	472	598	615	1,710

2. Match each table to an equation that represents the relationship.

   - Equation 1: $400 + x = y$
   - Equation 2: $x - 400 = y$
   - Equation 3: $x + y = 400$
   - Equation 4: $400 \cdot x = y$

Are you ready for more?

Express as many whole numbers between 1 and 20 as you can at least one way using exactly four 4's and any operation or mathematical symbol. For example, 1 could be written as $\frac{4}{4} + 4 - 4$.

3.3 What Are the Relationships?

1. The table represents the relationship between the base length and the height of some parallelograms. Both measurements are in inches.

base length (inches)	height (inches)
1	48
2	24
3	16
4	12
6	8

   What is the relationship between the base length and the height of these parallelograms?

2. Visitors to a carnival are invited to guess the number of beans in a jar. The person who guesses the correct number wins $300. If multiple people guess correctly, the prize will be divided evenly among them.

   What is the relationship between the number of people who guess correctly and the amount of money each person will receive?

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3. A $\frac{1}{2}$-gallon jug of milk can fill 8 cups, while 32 fluid ounces of milk can fill 4 cups.

   What is the relationship between the number of gallons and ounces? If you get stuck, try creating a table.

Lesson 3 Summary

Sometimes, the relationship between two quantities is easy to see. For instance, we know that the perimeter of a square is always 4 times the side length of the square. If P represents the perimeter and s represents the side length, then the relationship between the two measurements (in the same unit) can be expressed as $P = 4s$, or $s = \frac{P}{4}$.

Other times, the relationship between quantities might take a bit of work to figure out—by doing calculations several times or by looking for a pattern. Here are two examples.

- A plane departed from New Orleans and is heading to San Diego. The table shows its distance from New Orleans, x, and its distance from San Diego, y, at some points along the way.

miles from New Orleans	miles from San Diego
100	1,500
300	1,300
500	1,100
	1,020
900	700
1,450	
x	y

  What is the relationship between the two distances? Do you see any patterns in how each quantity is changing? Can you find out what the missing values are?

Notice that every time the distance from New Orleans increases by some number of miles, the distance from San Diego decreases by the same number of miles, and that the sum of the two values is always 1,600 miles.

The relationship can be expressed with any of these equations:

$$x + y = 1{,}600 \qquad y = 1,600 - x \qquad x = 1,600 - y$$

- A company decides to donate $50,000 to charity. It will select up to 20 charitable organizations, as nominated by its employees. Each selected organization will receive an equal amount of donation.

  What is the relationship between the number of selected organizations, n, and the dollar amount each of them will receive, d?

  - If 5 organizations are selected, each one receives $10,000.
  - If 10 organizations are selected, each one receives $5,000.
  - If 20 organizations are selected, each one receives $2,500.

  Do you notice a pattern here? 10,000 is $\frac{50{,}000}{5}$, 5,000 is $\frac{50{,}000}{10}$, and 2,500 is $\frac{50{,}000}{20}$.

  We can generalize that the amount each organization receives is 50,000 divided by the number of selected organizations, or $d = \frac{50{,}000}{n}$.