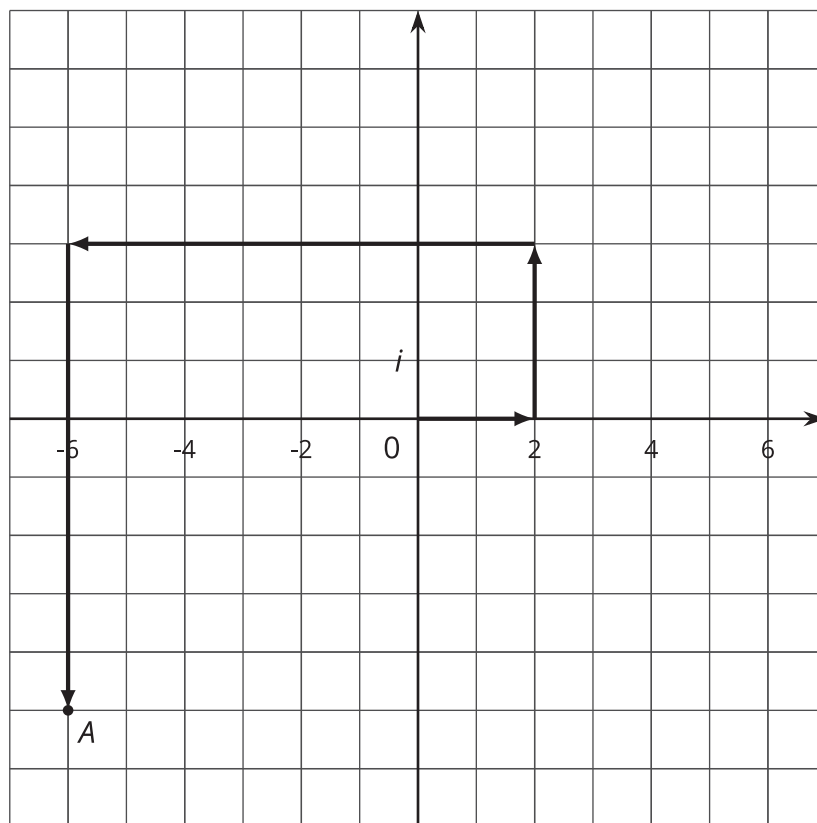


# Arithmetic with Complex Numbers

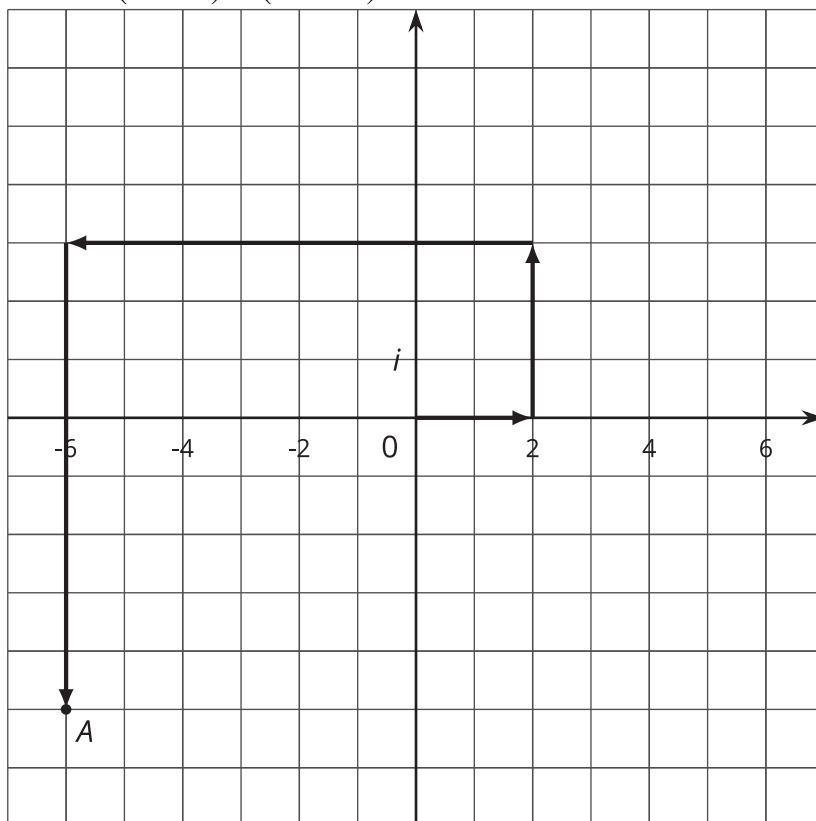
Let's work with complex numbers.

## 12.1 What Does This Path Mean?



## 12.2 Adding Complex Numbers

1. This diagram represents  $(2 + 3i) + (-8 - 8i)$ .



- How do you see  $2 + 3i$  represented?
- How do you see  $-8 - 8i$  represented?
- What complex number does  $A$  represent?
- Add “like terms” in the expression  $(2 + 3i) + (-8 - 8i)$ . What do you get?

2. Write these sums and differences in the form  $a + bi$ , where  $a$  and  $b$  are real numbers.

a.  $(-3 + 2i) + (4 - 5i)$  (Check your work by drawing a diagram.)

b.  $(-37 - 45i) + (11 + 81i)$

c.  $(-3 + 2i) - (4 - 5i)$

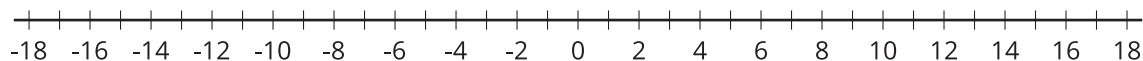
d.  $(-37 - 45i) - (11 + 81i)$



## 12.3

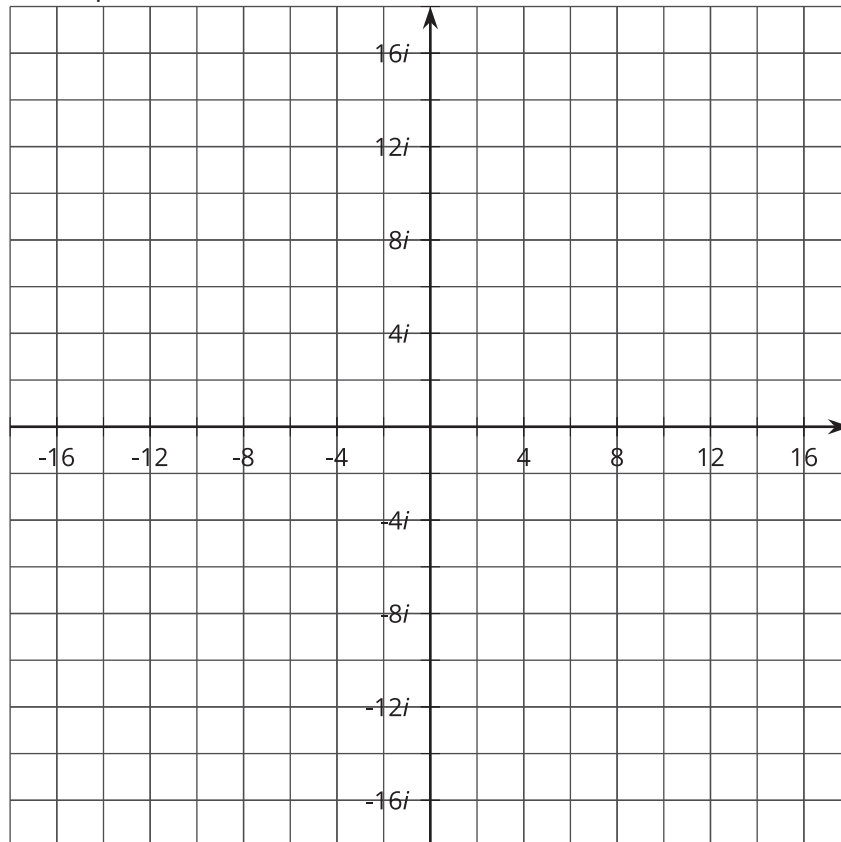
# Multiplication on the Complex Plane

1. Draw points to represent  $2$ ,  $2^2$ ,  $2^3$ , and  $2^4$  on the real number line. What do you notice about the arrangement of the points?



2. a. Write  $2i$ ,  $(2i)^2$ ,  $(2i)^3$ , and  $(2i)^4$  in the form  $a + bi$ .

- b. Plot  $2i$ ,  $(2i)^2$ ,  $(2i)^3$ , and  $(2i)^4$  on the complex plane. What do you notice about the arrangement of the points?





 **Are you ready for more?**

1. If  $a$  and  $b$  are positive numbers, is it true that  $\sqrt{ab} = \sqrt{a}\sqrt{b}$ ? Explain how you know.
2. If  $a$  and  $b$  are negative numbers, is it true that  $\sqrt{ab} = \sqrt{a}\sqrt{b}$ ? Explain how you know.



## Lesson 12 Summary

When we add a real number with an imaginary number, we get a complex number. We usually write complex numbers as:

$$a + bi$$

where  $a$  and  $b$  are real numbers. We say that  $a$  is the “real part” and  $b$  is the “imaginary part.”

To add (or subtract) two complex numbers, we add (or subtract) the real parts and add (or subtract) the imaginary parts. For example:

$$(2 + 3i) + (4 + 5i) = (2 + 4) + (3i + 5i) = 6 + 8i$$

$$(2 + 3i) - (4 + 5i) = (2 - 4) + (3i - 5i) = -2 - 2i$$

In general:

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

and:

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

When we raise an imaginary number to a power, we can use the fact that  $i^2 = -1$  to write the result in the form  $a + bi$ . For example,  $(4i)^3 = 4i \cdot 4i \cdot 4i$ . We can group the  $i$  factors together to see how to rewrite this.

$$\begin{aligned} 4i \cdot 4i \cdot 4i &= (4 \cdot 4 \cdot 4) \cdot (i \cdot i \cdot i) \\ &= 64 \cdot (i^2 \cdot i) \\ &= 64 \cdot -1 \cdot i \\ &= -64i \end{aligned}$$