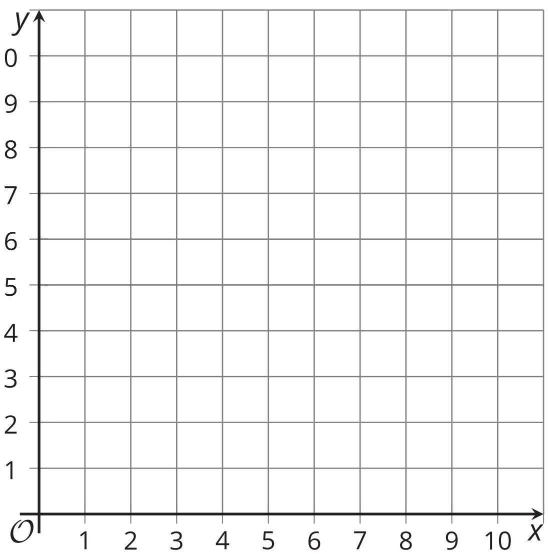
Unit 2, Lesson 12

Weighted Averages

Let's split segments using averages and ratios.

12.1 Part Way: Points

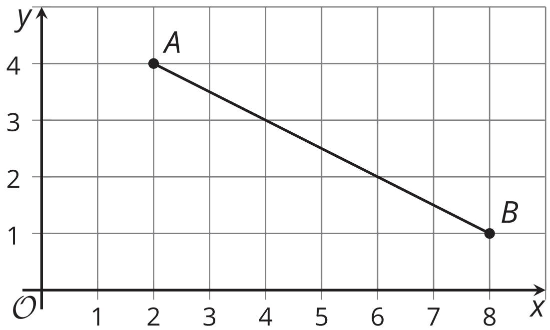
For the questions in this activity, use the coordinate grid if it is helpful to you.



1. What is the midpoint of the segment connecting $(1, 2)$ and $(5, 2)$?
2. What is the midpoint of the segment connecting $(5, 2)$ and $(5, 10)$?
3. What is the midpoint of the segment connecting $(1, 2)$ and $(5, 10)$?

12.2 Part Way: Segment

Point A has coordinates $(2,4)$. Point B has coordinates $(8,1)$.



1. Find the point that partitions segment AB in a $2:1$ ratio.
2. Calculate $C=\frac{1}{3}A+\frac{2}{3}B$.
3. What do you notice about your answers to the first 2 questions?
4. For 2 new points K and L, write an expression for the point that partitions segment KL in a $3:1$ ratio.

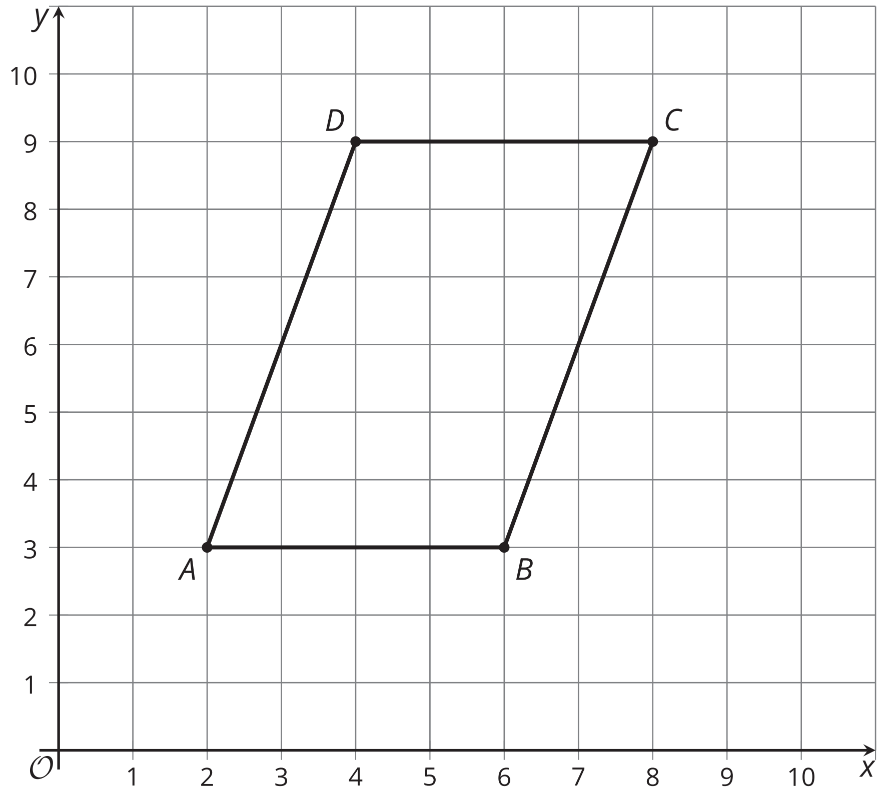
Are you ready for more?

Consider the general quadrilateral $QRST$ with $Q(0,0)$, $R(a,b)$, $S(c,d)$, and $T(e,f)$.

1. Find the midpoints of each side of this quadrilateral.
2. Show that if these midpoints are connected consecutively, the new quadrilateral formed is a parallelogram.

12.3 Part Way: Quadrilateral

Here is quadrilateral $ABCD$.



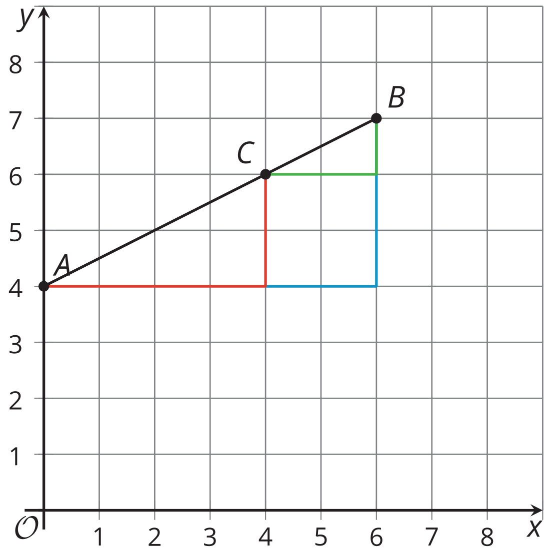
1. Find the point that partitions segment AB in a $1:4$ ratio. Label it B'.
2. Find the point that partitions segment AD in a $1:4$ ratio. Label it D'.
3. Find the point that partitions segment AC in a $1:4$ ratio. Label it C'.
4. Is $AB'C'D'$ a dilation of $ABCD$? Justify your answer.

Lesson 12 Summary

To find the midpoint of a line segment, we can average the coordinates of the endpoints. For example, to find the midpoint of the segment from $A(0,4)$ to $B(6,7)$, average the coordinates of A and B: $\left(\frac{0+6}{2}, \frac{4+7}{2}\right) = (3, 5.5)$. Another way to write what we just did is $\frac{1}{2}(A+B)$ or $\frac{1}{2}A + \frac{1}{2}B$.

Now, let's find the point that is $\frac{2}{3}$ of the way from A to B. In other words, we'll find point C so that segments AC and CB are in a $2:1$ ratio.

In the horizontal direction, segment AB stretches from $x=0$ to $x=6$. The distance from 0 to 6 is 6 units, so we calculate $\frac{2}{3}$ of 6 to get 4. Point C will be 4 horizontal units away from A, which means an x-coordinate of 4.



In the vertical direction, segment AB stretches from $y=4$ to $y=7$. The distance from 4 to 7 is 3 units, so we can calculate $\frac{2}{3}$ of 3 to get 2. Point C must be 2 vertical units away from A, which means a y-coordinate of 6.

It is possible to do this all at once by saying $C=\frac{1}{3}A+\frac{2}{3}B$. This is called a weighted average. Instead of finding the point in the middle, we want to find a point closer to B than to A. So we give point B more weight—it has a coefficient of $\frac{2}{3}$ rather than $\frac{1}{2}$ as in the midpoint calculation. To calculate $C=\frac{1}{3}A+\frac{2}{3}B$, substitute and evaluate.

$$\frac{1}{3}A+\frac{2}{3}B$$

$$\frac{1}{3}(0,4)+\frac{2}{3}(6,7)$$

$$\left(0,\frac{4}{3}\right)+\left(4,\frac{14}{3}\right)$$

$$(4,6)$$

Either way, we found that the coordinates of C are $(4,6)$.