

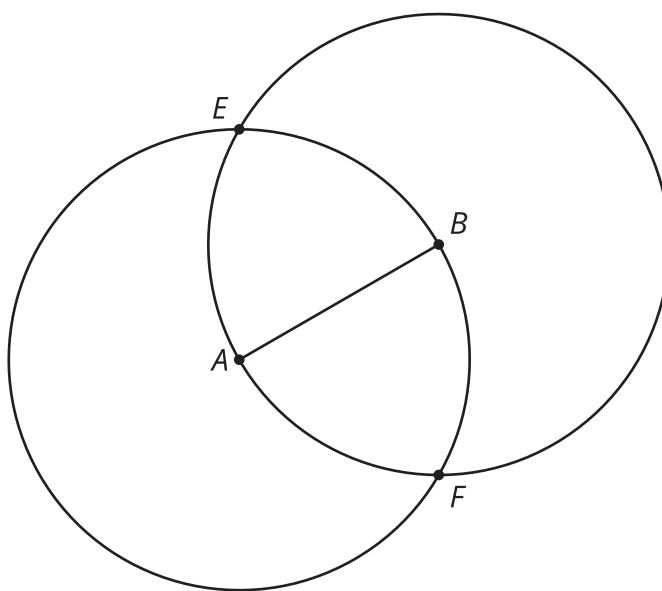


# Construction Techniques 3: Perpendicular Lines and Angle Bisectors

Let's use tools to solve some construction challenges.

## 5.1 Two Circles

Points  $A$  and  $B$  are each at the centers of circles of radius  $AB$ .



1. Compare the distance  $EA$  to the distance  $EB$ . Be prepared to explain your reasoning.
2. Compare the distance  $FA$  to the distance  $FB$ . Be prepared to explain your reasoning.
3. Draw line  $EF$ , and write a conjecture about its relationship with segment  $AB$ .



## 5.2 Make It Right

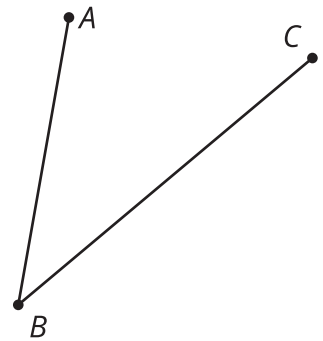
Here is a line  $\ell$  with a point labeled  $C$ . Use straightedge and compass moves to construct a line perpendicular to  $\ell$  that goes through  $C$ .



## 5.3 Bisect This

Here is an angle:

1. Estimate the location of a point  $D$  so that angle  $ABD$  is approximately congruent to angle  $CBD$ .
2. Use compass and straightedge moves to create a ray that divides angle  $CBA$  into 2 congruent angles. How close is the ray to going through your point  $D$ ?

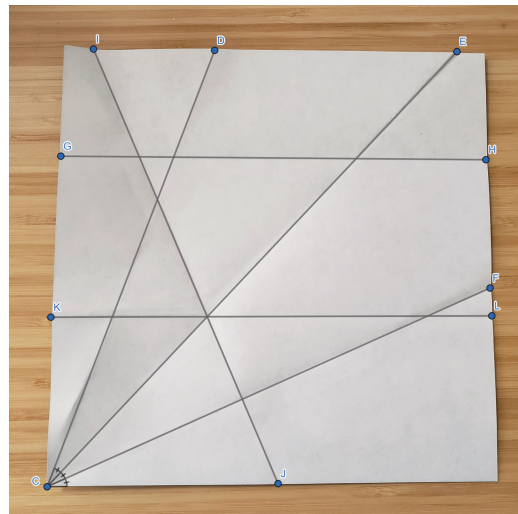


3. Take turns with your partner drawing and bisecting other angles.
  - a. For each angle that you draw, explain to your partner how each straightedge and compass move helps you to bisect it.
  - b. For each angle that your partner draws, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

### Are you ready for more?

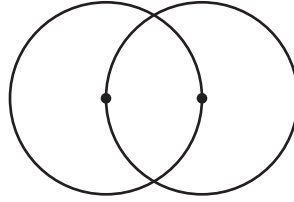
For thousands of years, people strived to find a construction to trisect an arbitrary angle into three equal angles. Many claimed to have found such a construction, but there was always some flaw in their reasoning. Finally, in 1837, French mathematician Pierre Wantzel used a new field of mathematics to prove it was impossible—which still did not stop some from claiming to have found a construction. If we allow other tools besides just a straightedge and compass, though, it is possible. For example, try this method of using origami (paper folding) to trisect an angle.

Video 'Trisecting an Angle with Origami' available here: <https://player.vimeo.com/video/298418799>.

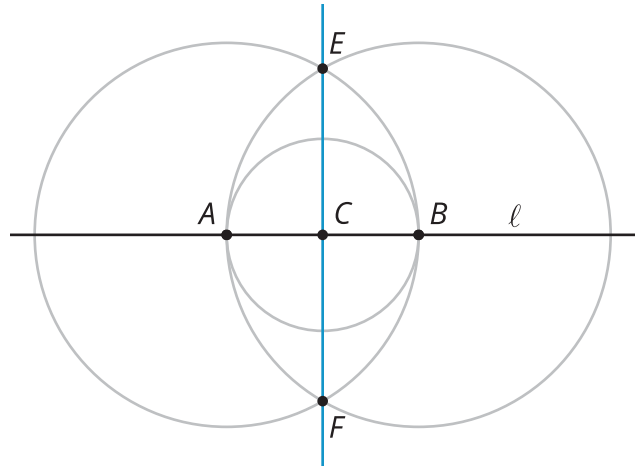


## Lesson 5 Summary

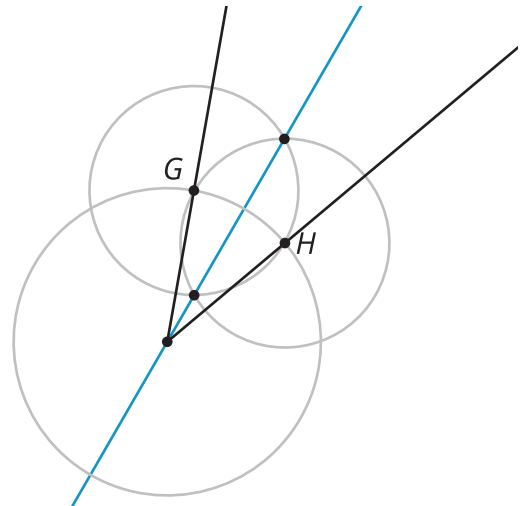
We can construct a line that is perpendicular to a given line. We can also bisect a given angle using only a straightedge and compass. The line that goes through the vertex of an angle to divide it into two equal angles is called the **angle bisector**. Both constructions use 2 circles that go through each other's centers:



To construct a line perpendicular to line  $\ell$  that goes through a given point  $C$ , start by finding 2 points, labeled here as  $A$  and  $B$ , on the given line  $\ell$  that are the same distance from  $C$ . Then create 2 circles of the same size centered at  $A$  and  $B$  that go through each other's centers. Connect the intersection points of those circles to draw a perpendicular line,  $EF$ .



To construct an angle bisector, start by finding 2 points, labeled here as  $G$  and  $H$ , that are on the rays and the same distance from the vertex. Then create the 2 circles of the same size centered at  $G$  and  $H$  that go through each other's centers. Connect the intersection points of those circles to draw the angle bisector.



In fact, we can think of creating a perpendicular line as bisecting a 180 degree angle!