



Systems of Linear Equations and Their Solutions

Goals

- Determine whether a system of equations will have 0, 1, or infinitely many solutions by analyzing their structure or by graphing.
- Recognize that a system of linear equations can have 0, 1, or infinitely many solutions.
- Use the structure of the equations in a linear system to make sense of the properties of their graphs.

Learning Targets

- I can tell how many solutions a system has by graphing the equations or by analyzing the parts of the equations and considering how they affect the features of the graphs.
- I know the possibilities for the number of solutions a system of equations could have.

Lesson Narrative

This lesson serves two main goals. The first goal is to revisit the idea (first learned in middle school) that not all systems of linear equations have a single solution. Some systems have no solutions and others have infinitely many solutions. The second goal is to investigate different ways to determine the number of solutions to a system of linear equations.

Earlier in the unit, students learned that the solution to a system of equations is a pair of values that meet both constraints in a situation, and that this condition is represented by a point of intersection of two graphs. Here, students make sense of a system with no solutions in a similar fashion. They interpret it to mean that there is no pair of values that meet both constraints in a situation, and that there is no point at which the graph of each equation would intersect.

Next, students use what they learned about the structure of equations and about equivalent equations to reason about the number of solutions. For instance, students recognize that equivalent equations have the same solution set. This means that if the two equations in a system are equivalent, we can tell—without graphing—that the system has infinitely many solutions. These exercises are opportunities to look for and make use of structure (MP7).

Likewise, students are aware that the graphs of linear equations with the same slope but different vertical intercepts are parallel lines. If the equations in a system can be rearranged into slope-intercept form (where the slope and vertical intercept become "visible"), it is possible to determine how many solutions a system has without graphing.

Standards

Building On 8.EE.8, A-CED.4
Addressing A-CED.3, A-REI.6

Instructional Routines

- Card Sort
- Graph It
- MLR6: Three Reads
- MLR8: Discussion Supports



Required Materials

Materials to Gather

- Graphing technology: Activity 1, Activity 2

Materials to Copy

- Sorting Systems Cards (1 copy for every 2 students): Activity 3

Required Preparation


Activity 1:

Acquire devices that can run Desmos (recommended) or other graphing technology. It is ideal if students each have their own device. (Desmos is available under Math Tools.)

Activity 2:

Acquire devices that can run Desmos (recommended) or other graphing technology. It is ideal if students each have their own device. (Desmos is available under Math Tools.)

Student Facing Learning Goals

 Let's find out how many solutions a system of equations could have.

17.1

A Curious System

Warm-up

 10 min

Activity Narrative

This *Warm-up* reminds students about the fact that some systems of linear equations have many solutions, while also prompting them to use what they learned in this unit to better understand what it means for a system to have infinitely many solutions.

The first equation shows two variables adding up to 5, so students choose a pair of values whose sum is 5. They notice that all pairs chosen are solutions to the system. Next, they try to find a strategy that can show that there are countless other pairs that also satisfy the constraints in the system. Monitor for these likely strategies:

- Solving by graphing: The graphs of the two equations are the same line, so all the points on the line are solutions to the system.
- Solving by substitution: The first equation can be rearranged to $y = 5 - x$. Substituting $5 - x$ for y in the second equation gives $4x = 20 - 4(5 - x)$, or $4x = 20 - 20 + 4x$, or $4x = 4x$. This equation is true no matter what x is.
- Solving by elimination: If we multiply the first equation by 4, rearrange the second equation to $4x + 4y = 20$, and then subtract the second equation from the first, the result is $4y = 4y$. Subtracting $4y$ from each side gives $0 = 0$, which is true regardless of what x or y is.
- Reasoning about equivalent equations: If we rearrange the second equation so that the variables are on the same side $4x + 4y = 20$, we can see that this equation is a multiple of the first and that the two equations are equivalent. This means they have the exact same solution set, which contains infinite possible pairs of x and y .

Identify students who use different strategies, and ask them to share their thinking with the class later.

Making graphing technology available gives students an opportunity to choose appropriate tools strategically (MP5).



Launch

Arrange students in groups of 3–4. Give students 1–2 minutes of quiet time to complete the first two questions. Remind students that the numbers don't have to be integers. Then, give students a minute to share with their group what their two values are and whether the pair is a solution to the second equation.

Pause for a brief whole-class discussion. Consider recording quickly all the pairs that students have verified to be solutions to the second equation and displaying them for all to see. Ask: "Which two numbers are the solution to the system?" (All of them)

Ask students to proceed to the last question. Students may conclude that seeing all the pairs that satisfy both equations is enough to show that the system has many solutions. If so, ask how many solutions they think there are, and ask "Can you show that *any* two numbers that add up to 5 is also a solution to the system, without having to do calculations for each pair?"

Student Task Statement

Andre is trying to solve this system of equations:
$$\begin{cases} x + y = 5 \\ 4x = 20 - 4y \end{cases}$$

Looking at the first equation, he thought, "The solution to the system is a pair of numbers that add up to 5. I wonder which two numbers they are."

1. Choose any two numbers that add up to 5. Let the first one be the x -value and the second one be the y -value.
2. The pair of values you chose is a solution to the first equation. Check if it is also a solution to the second equation. Then, pause for a brief discussion with your group.
3. How many solutions does the system have? Use what you know about equations or about solving systems to show that you are right.

Student Response

1. Sample responses: 6 and -1, 0 and 5, $\frac{1}{2}$ and $4\frac{1}{2}$
2. Yes, it is also a solution to the second equation. Sample reasoning for 6 and -1: Substituting 6 and -1 into the second equation gives $4(6) = 20 - 4(-1)$, or $24 = 20 + 4$, which is a true equation.
3. Infinitely many solutions. See the *Activity Narrative* for some possible explanations.

Activity Synthesis

Ask the class how many solutions they think the system has. Select previously identified students to explain how they know that the system has infinitely many solutions, or that any pair of values that add up to 5 and make the first equation true also make the second equation true.

Students are likely to think of using graphs. It is not essential to elicit all strategies shown in the *Activity Narrative*, but if no one thinks of an algebraic explanation, be sure to bring one up. The last strategy mentioned in the *Activity Narrative*

(reasoning about equivalent equations) is likely to be intuitive to students.

Make sure students see that the equations are equivalent, so all pairs of x and y values that make one equation true also make the other equation true, giving an infinite number of solutions.

17.2 What's the Deal?

🕒 10 min

Activity Narrative

In the *Warm-up*, students saw that some systems have infinitely many solutions. In this activity, they encounter a situation that can be represented with a system of equations but the system has no solutions. Students write equations to represent the two constraints in the situation and then solve the system algebraically and graphically.

As students work, notice the different ways in which they reach the conclusion that the systems have no solutions. Identify students with varying strategies, and ask them to share later.

Standards

Building On 8.EE.8
Addressing A-CED.3, A-REI.6

Instructional Routines

- Graph It
- MLR6: Three Reads

Launch

Keep students in groups of 3–4 and provide access to graphing technology.

Access for English Language Learners

MLR6 Three Reads. Keep books or devices closed. Display only the image and problem stem, without revealing the questions. “We are going to read this situation 3 times.”

- After the 1st read, say “Tell your partner what this situation is about.”
- After the 2nd read, say “List the quantities. What can be counted or measured?”
- For the 3rd read, reveal and read the questions. Ask “What are some ways we might get started on this?”

Advances: Reading, Representing

Access for Students with Disabilities

Representation: Access for Perception. Read the scenario aloud. Students who both listen to and read the information will benefit from extra processing time. Encourage students to annotate their document as they follow along.

Supports accessibility for: Language, Attention

Student Task Statement

A recreation center is offering special prices on its pool passes and gym memberships for the summer. On the first day of the offering, a family paid \$96 for 4 pool passes and 2 gym memberships. Later that day, an

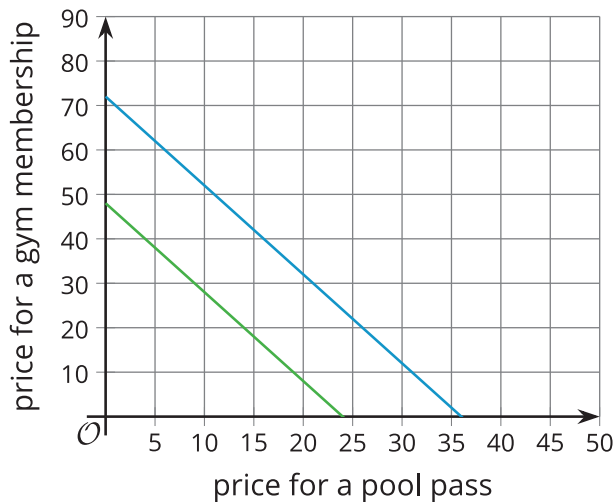


individual bought a pool pass for herself, a pool pass for a friend, and 1 gym membership. She paid \$72.

1. Write a system of equations that represents the relationships between pool passes, gym memberships, and the costs. Be sure to state what each variable represents.
2. Find the price of a pool pass and the price of a gym membership by solving the system algebraically. Explain or show your reasoning.
3. Use graphing technology to graph the equations in the system. Make 1-2 observations about your graphs.

Student Response

1. $\begin{cases} 4p + 2g = 96 \\ 2p + 1g = 72 \end{cases}$, where p is the price of each pool pass and g is the price of gym membership for 1 person.
2. The system has no solutions. Sample reasoning: Solving by elimination or substitution leads to a false equation (such as $0 = 24$).



Sample observations: The graphs are two parallel lines that don't intersect. The lines have the same slope but different vertical intercepts.

3.

Activity Synthesis

Invite previously identified students to share their response to the second question. Record or display their reasoning for all to see. After each student shares, ask if anyone else reasoned the same way.

Next, select other students to share their observations about the graphs. Ask students:

- “How can we tell from the graphs that there are no solutions?” (The lines are parallel.)
- “How can we tell for sure that the lines are parallel and never intersect?” (The slope of both graphs is -2 but they have different intercepts.)
- “Why do parallel lines mean no solutions?” (A solution is a pair of values that satisfy both equations and are on both graphs. There are no points that are on both lines simultaneously.)
- “What does ‘no solutions’ mean in this situation, in terms of price of pool passes and gym memberships?” (The prices for a pool pass and for a gym membership are different for the two purchases.)

Here are some ways to think about the situation:

- The family purchased twice the number of pool passes and gym memberships as the individual did, but they did not pay twice as much, so the prices of passes and memberships must have been different for the two purchases.
- The person who bought half as many passes and memberships did not pay half as much, which meant that



different prices applied to the two transactions.

- The special rates for a family of 4 did not apply to individuals, hence the different prices.

17.3

Card Sort: Sorting Systems

🕒 15 min

Activity Narrative

In earlier activities, students gained some insights into the structure of equations in systems that have infinitely many solutions and those that have no solutions. In this activity, they apply those insights to sort systems of equations based on the number of solutions (one solution, many solutions, or no solutions).

Students could solve each system algebraically or graphically and sort afterward, but given the number of systems to be solved, they will likely find this process to be time consuming. A more productive way would be to look for and make use of the structure in the equations in the systems (MP7), for example, by looking out for equivalent equations, equations with the same slope but different vertical intercepts, variable expressions with the same or opposite coefficients, and so on.

To effectively make use of the structure of the systems, students need to attend closely to all parts of each equation—the signs, variables, coefficients, and constants—and to rearrange equations with care (MP6).

As students discuss their thinking in groups, make note of the different ways they use structure to complete the task. Encourage students who are solving individual systems to analyze the features of the equations and see if they could reason about the solutions or gain information about the graphs that way.

In this activity, students are analyzing the structure of equations in the systems, so technology is not an appropriate tool.

Standards

Building On A-CED.4
Addressing A-REI.6

Instructional Routines

- Card Sort
- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Give one set of pre-cut slips or cards from the blackline master to each group.

Give students 7–8 minutes to sort the cards into groups. Emphasize to students that they should be prepared to explain how they place each system. Follow with a whole-class discussion.

Access for English Language Learners

MLR8 Discussion Supports. Students should take turns selecting a card and explaining their reasoning to their partner. Display the following sentence frames for all to see: “I noticed ____, so I matched” Encourage students to challenge each other when they disagree.

Advances: Conversing

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Give students a subset of the cards to start with, and introduce the remaining cards after students have completed their initial set of matches. Be sure to include at least one card from each category in the initial set.

Supports accessibility for: Conceptual Processing, Organization, Memory

Student Task Statement

Your teacher will give you a set of cards. Each card contains a system of equations.

Sort the systems into three groups based on the number of solutions each system has. Be prepared to explain how you know where each system belongs.

Student Response

- No solutions: Cards 2 and 5
- One solution: Cards 1, 4, 6, and 9
- Infinitely many solutions: Cards 3, 7, and 8

Building on Student Thinking

Some students may not know how to begin sorting the cards. Suggest that they try solving 2–3 systems. Ask them to notice if there's a point in the solving process when they realize how many solutions the system has or what the graphs of the two equations would look like. Encourage students to look for similarities in the structure of the equations and to see how the structure might be related to the number of solutions.

Are You Ready for More?

1. In the cards, for each system with no solution, change a single constant term so that there are infinitely many solutions to the system.
2. For each system with infinitely many solutions, change a single constant term so that there are no solutions to the system.
3. Explain why in these situations it is impossible to change a single constant term so that there is exactly one solution to the system.

Extension Student Response

Sample responses:

1. Card 2: change 3 to 13, Card 5: change -12 to 12
2. Any change to a constant term would work.
3. Changing a constant term won't change the slope of the lines, so the graphs will either be distinct parallel lines or the same line.

Activity Synthesis

Invite groups to share their sorting results, and record them. Ask the class if they agree or disagree. If there are

disagreements, ask students who disagree to share their reasoning.

Display all the systems—sorted into groups—for all to see, and discuss the characteristics of the equations in each group. Ask students questions such as:

- “How can we tell from looking at the equations in cards 2 and 5 that the systems have no solution?” (Possible reasoning:
 - Card 2: The equations are in slope-intercept form. The slope is 2 for both graphs, but the y -intercept is different, so the lines must be parallel.
 - Card 5: The second equation can be rearranged into $5x - 20y = 60$ and multiplied by $\frac{1}{5}$ to give $x - 4y = 12$. Both equations now have $x - 4y$ on one side, but that expression is equal to -12 in the first equation and equal to 12 in the second. There is no pair of x and y that can make both equations true.)
- “What about the equations in cards 3, 7, and 8? What features might give us a clue that the systems have many solutions?” (Possible reasoning:
 - Card 3: The coefficients and constants in the second equation are 3 times those in the first, so they are equivalent equations.
 - Card 7: The first equation can be rearranged into the same form as the first: $x - 4y = -4$. We can then see that the second equation, $4x - 16y = -16$, is a multiple of the first equation, so they have all the same solutions.
 - Card 8: The first equation can be rearranged to $y = -25x - \frac{1}{5}$. We can then see that it is related to the second equation by a factor of 5, so they are equivalent equations.)

We can reason that all the other systems have one solution by a process of elimination—by noticing that they don’t have the features of systems with many solutions or systems with no solutions.

Lesson Synthesis

To help students summarize and organize the insights they gained in the lesson, consider asking them to complete (collaboratively in small groups or as a class) a graphic organizer with the following components for each type of system (no solutions, many solutions, and one solution).



A System with _____ (One, No, Many) Solution(s)

<p>Meaning: What does it mean for a system to have (one, no, many) solution(s)?</p>	<p>Equations: What are some characteristics of the equations in such system? (Give an example, if possible.)</p>
<p>Graphs: What are some characteristics of the graphs of the equations in the system? (Sketch an example, if possible.)</p>	<p>Solutions: What would we get when solving the system of equations algebraically? (Give an example, if possible.)</p>

If time is limited, focus on describing some characteristics of the equations in a system with each number of solutions (one, many, none) and sketching their graphs. Consider using a graphic organizer such as this one.

	<p>Graphs: What are some characteristics of the graphs of the equations in the system? (Sketch an example, if possible.)</p>	<p>Solutions: What do we get when solving the system of equations algebraically? (Give an example, if possible.)</p>
<p>One Solution</p>		
<p>Many Solutions</p>		
<p>No Solutions</p>		

17.4

No Graphs, No Problem

Cool-down

🕒 5 min

Graphing technology should not be used in this *Cool-down*.

If time is limited, ask students to choose one system and to explain how they could tell that it has no solutions or infinitely many solutions.



Standards

Addressing A-REI.6

Student Task Statement

Mai is given these two systems of linear equations to solve:

System 1:

$$\begin{cases} 5x + y = 13 \\ 20x + 4y = 64 \end{cases}$$

System 2:

$$\begin{cases} 5x + y = 13 \\ 20x = 52 - 4y \end{cases}$$

She analyzed them for a moment, and then—without graphing the equations—said, "I got it! One of the systems has no solution, and the other has infinitely many solutions!" Mai is right!

Which system has no solution, and which one has many solutions? Explain or show how you know (without graphing the equations).

Student Response

The first system has no solutions, and the second system has infinitely many solutions. Sample explanations:

- Multiplying the first equation by 4 gives $20x + 4y = 52$. Subtracting the second equation from the first equation gives $0 = -12$, which is a false equation and tells us that the system has no solutions.
- $20x + 4y$ (in the second equation) is 4 times $5x + y$ (in the first equation), but 64 is not 4 times 13. This means there is no pair of x - and y -values that could make both equations simultaneously true.
- If we isolate y in the first equation, we have $y = 13 - 5x$. If we do the same with the second equation, we have $4y = 64 - 20x$ or $y = 16 - 5x$. The graphs of the two equations have the same slope (-5), and they intersect the y -axis at different points (13 and 16), so the lines are parallel and have no points (solutions) in common.

The second system has infinitely many solutions. Sample explanations:

- Multiplying the first equation by 4 gives $20x + 4y = 52$. Rearranging the second equation so that the variables are on the left side also gives $20x + 4y = 52$. The two equations are identical, so they have the same solutions.
- The second equation can be rearranged to $20x + 4y = 52$. We can see that it is 4 times the first equation, which means the two equations, $5x + y = 13$ and $20x + 4y = 52$, are equivalent and have all the same solutions.
- If we isolate y in each equation, we'd have the exact same equation, $y = 13 - 5x$, so they have all the same solutions.

Responding to Student Thinking

Points to Emphasize

If most students struggle to recognize when a system of equations has zero or infinitely many solutions, consider using the optional activity referred to here as additional practice. Before students generate their own equations to create a system, provide several additional options that would result in 0, 1, or infinitely many solutions, and ask students to explain what they notice about each situation.

Integrated Math 1, Unit 4, Lesson 18, Activity 2 One, Zero, Infinitely Many





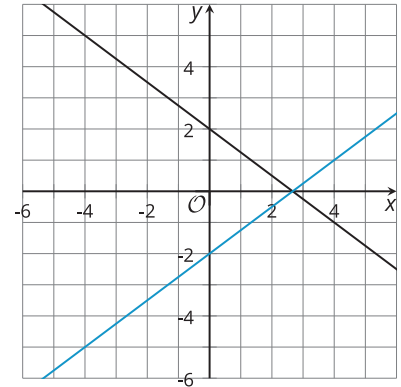
Lesson 17 Summary

We have seen many examples of a system in which one pair of values satisfies both equations. Not all systems, however, have one solution. Some systems have many solutions, and others have no solutions.

Let's look at three systems of equations and their graphs.

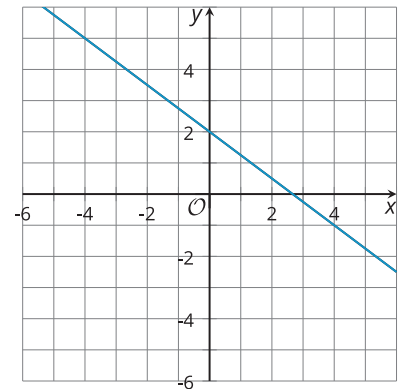
$$\text{System 1: } \begin{cases} 3x + 4y = 8 \\ 3x - 4y = 8 \end{cases}$$

The graphs of the equations in System 1 intersect at one point. The coordinates of the point are the one pair of values that are simultaneously true for both equations. When we solve the equations, we get exactly one solution.



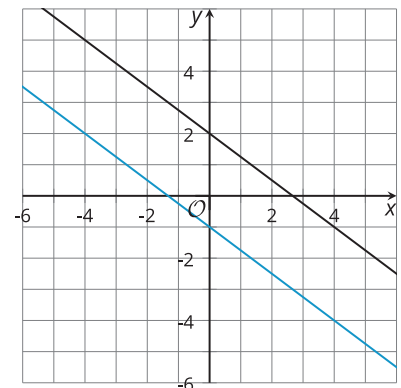
$$\text{System 2: } \begin{cases} 3x + 4y = 8 \\ 6x + 8y = 16 \end{cases}$$

The graphs of the equations in System 2 appear to be the same line. This suggests that every point on the line is a solution to both equations, or that the system has infinitely many solutions.



$$\text{System 3: } \begin{cases} 3x + 4y = 8 \\ 3x + 4y = -4 \end{cases}$$

The graphs of the equations in System 3 appear to be parallel. If the lines never intersect, then there is no common point that is a solution to both equations, and the system has no solutions.



How can we tell, without graphing, that System 2 indeed has many solutions?

- Notice that $3x + 4y = 8$ and $6x + 8y = 16$ are equivalent equations. Multiplying the first equation by 2 gives the second equation. Multiplying the second equation by $\frac{1}{2}$ gives the first equation. This means that any solution to the first equation is a solution to the second.
- Rearranging $3x + 4y = 8$ into slope-intercept form gives $y = \frac{8 - 3x}{4}$, or $y = 2 - \frac{3}{4}x$. Rearranging



$6x + 8y = 16$ gives $y = \frac{16 - 6x}{8}$, which is also $y = 2 - \frac{3}{4}x$. Both lines have the same slope and the same y -value for the vertical intercept!

How can we tell, without graphing, that System 3 has no solutions?

- Notice that in one equation $3x + 4y$ equals 8, but in the other equation it equals -4. Because it is impossible for the same expression to equal 8 and -4, there must not be a pair of x - and y -values that are simultaneously true for both equations. This tells us that the system has no solutions.
- Rearranging each equation into slope-intercept form gives $y = 2 - \frac{3}{4}x$ and $y = -1 - \frac{3}{4}x$. The two graphs have the same slope but the y -values of their vertical intercepts are different. This tells us that the lines are parallel and will never cross.



Lesson 17 Practice Problems

1 Student Task Statement

Here is a system of equations:
$$\begin{cases} 3x - y = 17 \\ x + 4y = 10 \end{cases}$$

- Solve the system by graphing the equations (by hand or using technology).
- Explain how you could tell, without graphing, that there is only one solution to the system.

Solution

- The solution is (6, 1).
- Sample explanations:
 - If we solve the system algebraically (by substitution or by elimination), we would find one value of x and one value of y that make both equations true.
 - If we rewrite both equations into slope-intercept form, we would have $y = 3x - 17$, and $y = -\frac{1}{4}x + \frac{5}{2}$. The graphs of the two equations have different slopes (3 and $-\frac{1}{4}$), and lines with different slopes can intersect at only one point.

2 Student Task Statement

Consider this system of linear equations:
$$\begin{cases} y = \frac{4}{5}x - 3 \\ y = \frac{4}{5}x + 1 \end{cases}$$

- Without graphing, determine how many solutions you would expect this system of equations to have. Explain your reasoning.
- Try solving the system of equations algebraically, and describe the result that you get. Does it match your prediction?

Solution

- No solutions. Sample reasoning: The two equations have the same coefficient for x and different vertical intercepts, so their graphs have the same slope and aren't the same line. This means that the lines never cross, and there is no (x, y) pair that is true for both equations.
- Sample response: Solving by substitution, I end up with the equation $-3 = 1$ (or some multiple of this equation), which is a false statement. This means that the system has no solutions, which matches my prediction.

3 Student Task Statement

How many solutions does this system of equations have? Explain how you know.





$$\begin{cases} 9x - 3y = -6 \\ 5y = 15x + 10 \end{cases}$$

Solution

Infinitely many solutions. Sample explanation: Both equations can be written as $y = 3x + 2$, so their graphs are exactly the same.

4 Student Task Statement

Select **all** systems of equations that have no solutions.

A. $\begin{cases} y = 5 - 3x \\ y = -3x + 4 \end{cases}$

B. $\begin{cases} y = 4x - 1 \\ 4y = 16x - 4 \end{cases}$

C. $\begin{cases} 5x - 2y = 3 \\ 10x - 4y = 6 \end{cases}$

D. $\begin{cases} 3x + 7y = 42 \\ 6x + 14y = 50 \end{cases}$

E. $\begin{cases} y = 5 + 2x \\ y = 5x + 2 \end{cases}$

Solution

A, D

5 from Unit 4, Lesson 16

Student Task Statement

Solve each system of equations without graphing.

a. $\begin{cases} 2v + 6w = -36 \\ 5v + 2w = 1 \end{cases}$

b. $\begin{cases} 6t - 9u = 10 \\ 2t + 3u = 4 \end{cases}$

Solution

a. (3, -7)

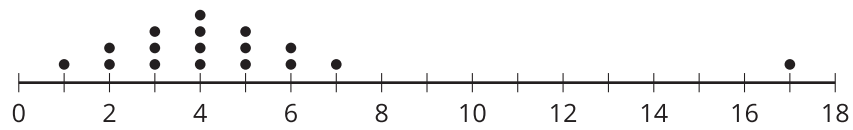
b. $(\frac{11}{6}, \frac{1}{9})$



 **Student Task Statement**

Select **all** the dot plots that appear to contain outliers.

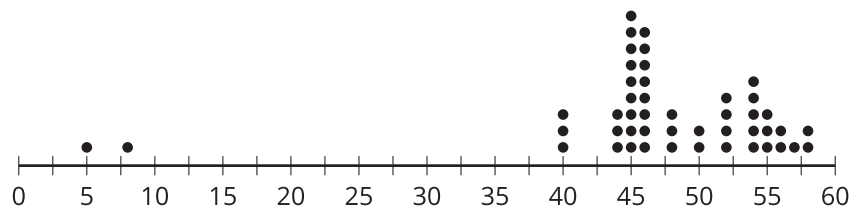
A.



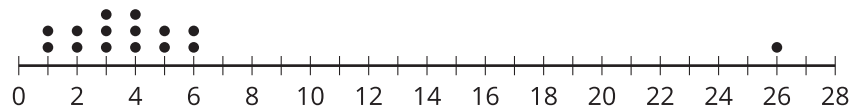
B.



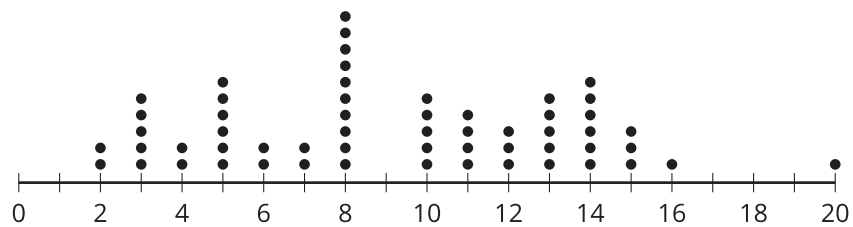
C.



D.



E.

**Solution**

A, C, D

 **Student Task Statement**

Here is a system of equations: $\begin{cases} -x + 6y = 9 \\ x + 6y = -3 \end{cases}$

Would you rather use subtraction or addition to solve the system? Explain your reasoning.

Solution

Sample responses:

- I would rather use addition because this eliminates x and allows us to solve for y .
- I would rather use subtraction because this eliminates y and allows us to solve for x .

8

from Unit 4, Lesson 16

Student Task Statement

Here is a system of linear equations:
$$\begin{cases} 6x - y = 18 \\ 4x + 2y = 26 \end{cases}$$

Select **all** the steps that would help to eliminate a variable and enable solving.

- A. Multiply the first equation by 2, then subtract the second equation from the result.
- B. Multiply the first equation by 4 and the second equation by 6, then subtract the resulting equations.
- C. Multiply the first equation by 2, then add the result to the second equation.
- D. Divide the second equation by 2, then add the result to the first equation.
- E. Multiply the second equation by 6, then subtract the result from the first equation.

Solution

B, C, D

9

from Unit 4, Lesson 16

Student Task Statement

Consider this system of equations, which has one solution:
$$\begin{cases} 2x + 2y = 180 \\ 0.1x + 7y = 78 \end{cases}$$

Here are some equivalent systems. Each one is a step in solving the original system.

Step 1:

$$\begin{cases} 7x + 7y = 630 \\ 0.1x + 7y = 78 \end{cases}$$

Step 2:

$$\begin{cases} 6.9x = 552 \\ 0.1x + 7y = 78 \end{cases}$$

Step 3:

$$\begin{cases} x = 80 \\ 0.1x + 7y = 78 \end{cases}$$

- a. Look at the original system and the system in Step 1.
 - i. What was done to the original system to get the system in Step 1?
 - ii. Explain why the system in Step 1 shares a solution with the original system.



- b. Look at the system in Step 1 and the system in Step 2.
 - i. What was done to the system in Step 1 to get the system in Step 2?
 - ii. Explain why the system in Step 2 shares a solution with that in Step 1.
- c. What is the solution to the original system?

Solution

- a. Sample response:
 - i. The first equation in the original system is multiplied by 3.5. The second equation stays the same.
 - ii. Multiplying both sides of the equation $2x + 2y = 180$ by the same number (3.5) keeps the two sides equal and gives an equivalent equation. The (x, y) pair that is a solution to $2x + 2y = 180$ is still a solution to $7x + 7y = 630$, so it is also a solution to the system in Step 1.
- b. Sample response:
 - i. Subtracting the second equation in Step 1 from the first equation gives the equation $6.9x = 552$ in Step 2. The second equation is unchanged.
 - ii. Because $0.1x + 7y$ is equal to 78, subtracting $0.1x + 7y$ from the left side of $7x + 7y = 630$ and 78 from the right side means subtracting equal amounts from both sides, which keeps the two sides equal. The (x, y) pair that is a solution to the system in Step 1 is also a solution to the resulting equation.
- c. $x = 80$ and $y = 10$, or $(80, 10)$

