



# Graphs of Rational Functions (Part 2)

Let's learn about horizontal asymptotes.

## 3.1 Rewritten Equations

Decide if each of these equations is true or false for  $x$ -values that do not result in a denominator of 0. Be prepared to explain your reasoning.

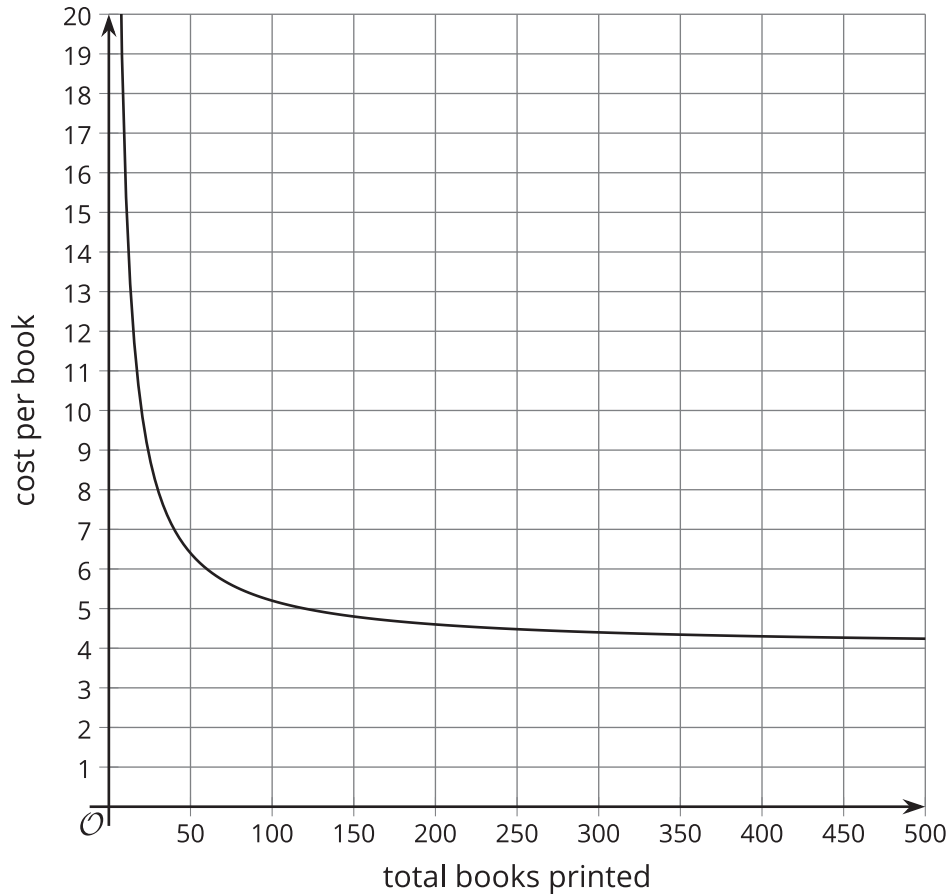
1.  $\frac{x+7}{x} = 1 + \frac{7}{x}$

2.  $\frac{x}{x+7} = 1 + \frac{x}{7}$



## 3.2 Publishing a Paperback

Let  $c$  be the function that gives the average cost per book  $c(x)$ , in dollars, when using an online store to print  $x$  copies of a self-published paperback book. Here is a graph of  $c(x) = \frac{120+4x}{x}$ .



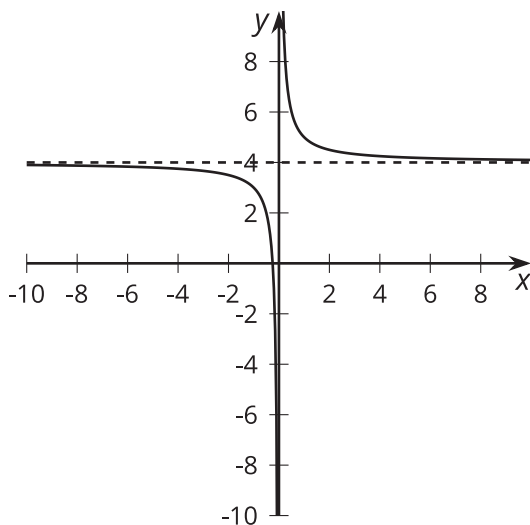
1. What is the approximate cost per book when 50 books are printed? 100 books?
2. The author plans to charge \$8 per book. About how many should be printed to make a profit?
3. What is the value of  $c(x)$  when  $x = \frac{1}{2}$ ? How does this relate to the context?
4. What does the end behavior of the function say about the context?



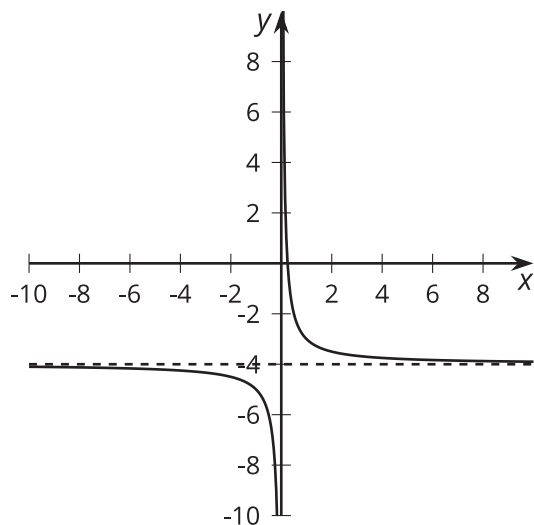
### 3.3 Horizontal Asymptotes

Here are four graphs of rational functions.

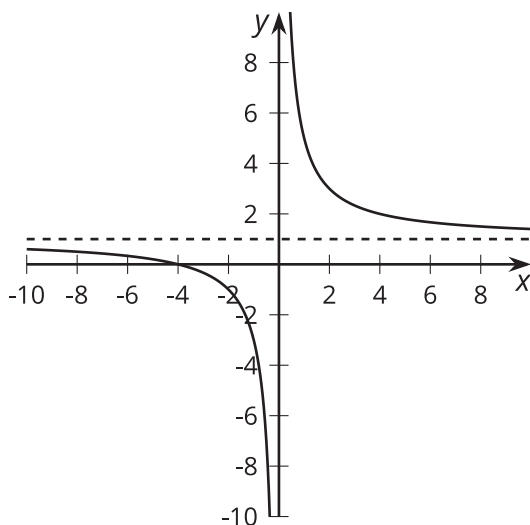
**A**



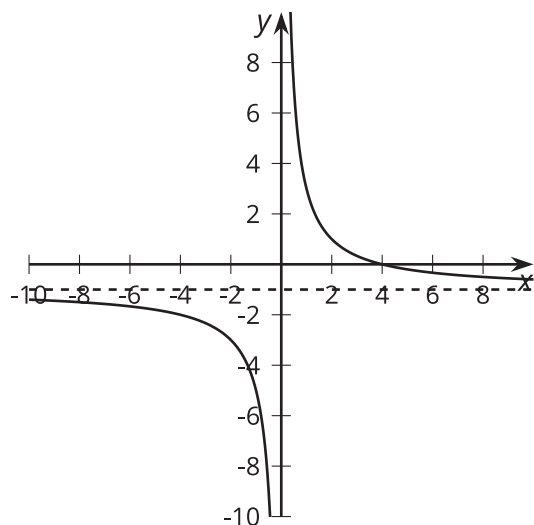
**B**



**C**



**D**



1. Match each function with its graphical representation.

◦  $a(x) = \frac{4}{x} - 1$

◦  $b(x) = \frac{1}{x} - 4$

◦  $c(x) = \frac{1+4x}{x}$

◦  $d(x) = \frac{x+4}{x}$

◦  $e(x) = \frac{1-4x}{x}$

◦  $f(x) = \frac{4-x}{x}$

◦  $g(x) = 1 + \frac{4}{x}$

◦  $h(x) = \frac{1}{x} + 4$



2. Where do you see the **horizontal asymptote** of the graph in the expressions for the functions?



**Are you ready for more?**

Consider the function  $a(x) = \frac{\frac{1}{2}x+1}{x-1}$ .

1. Predict where you think the vertical and horizontal asymptotes of  $a(x)$  will be. Explain your reasoning.
2. Use graphing technology to check your prediction.



### Lesson 3 Summary

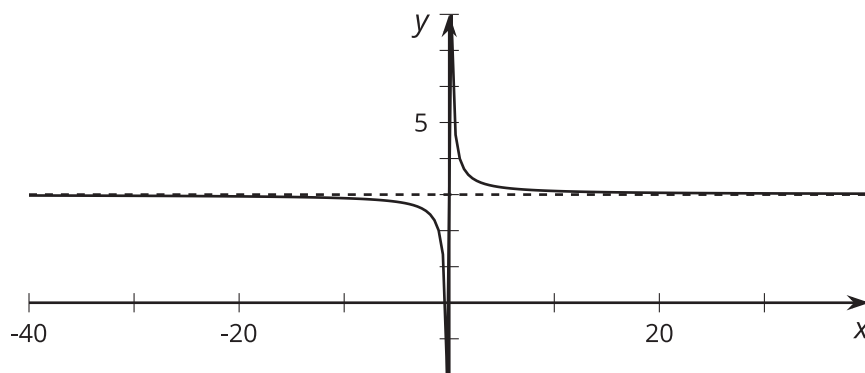
Consider the rational function  $f(x) = \frac{3x+1}{x}$ . Written this way, we can tell that the graph of the function has a vertical asymptote at  $x = 0$  by reading the denominator and identifying the value that would cause division by 0. But what can we tell about the value of  $f(x)$  for values of  $x$  far away from the vertical asymptote?

One way we can think about these values is to rewrite the expression for  $f(x)$  by breaking up the fraction:

$$f(x) = \frac{3x}{x} + \frac{1}{x}$$

$$f(x) = 3 + \frac{1}{x}$$

Written this way, it's easier to see that as  $x$  gets larger and larger in either the positive or negative direction, the  $\frac{1}{x}$  term will get closer and closer to 0. Because of this, we can say that the value of the function will get closer and closer to 3. Here is a graph of  $y = f(x)$  showing values from -40 to 40.



A dashed line at  $y = 3$  is included to show how the function approaches this value as inputs are farther and farther from  $x = 0$ . This is an example of a feature of rational functions: a horizontal asymptote.

The line  $y = c$  is a **horizontal asymptote** for a function if the value of the function gets closer and closer to  $c$  as the magnitude of  $x$  increases.

More generally, if a rational function  $g(x) = \frac{a(x)}{b(x)}$  can be rewritten as  $g(x) = c + \frac{r(x)}{b(x)}$ , where  $c$  is a constant and  $r(x)$  and  $b(x)$  are polynomial expressions in which  $\frac{r(x)}{b(x)}$  gets closer and closer to 0 as  $x$  gets larger and larger in both the positive and negative directions, then  $g(x)$  will get closer and closer to  $c$ .