



# Reasoning about Exponential Graphs (Part 1)

Let's study and compare equations and graphs of exponential functions

## 12.1 Spending Gift Money

Jada receives a gift of \$180. In the first week, she spends a third of the gift money. She continues spending a third of what is left each week thereafter. Which equation best represents the amount of gift money  $g$ , in dollars, she has after  $t$  weeks? Be prepared to explain your reasoning.

A.  $g = 180 - \frac{1}{3}t$

B.  $g = 180 \cdot \left(\frac{1}{3}\right)^t$

C.  $g = \frac{1}{3} \cdot 180^t$

D.  $g = 180 \cdot \left(\frac{2}{3}\right)^t$

## 12.2

## Equations and Their Graphs

1. Each of the functions  $f$ ,  $g$ ,  $h$ , and  $j$  represents the amount of money in a bank account, in dollars, as a function of time  $x$ , in years. They are each written in form  $m(x) = a \cdot b^x$ .

$$f(x) = 50 \cdot 2^x$$

$$g(x) = 50 \cdot 3^x$$

$$h(x) = 50 \cdot \left(\frac{3}{2}\right)^x$$

$$j(x) = 50 \cdot (0.5)^x$$

- Use graphing technology to graph each function on the same coordinate plane.
- Explain how changing the value of  $b$  changes the graph.

2. Here are equations defining functions  $p$ ,  $q$ , and  $r$ . They are also written in the form  $m(x) = a \cdot b^x$ .

$$p(x) = 10 \cdot 4^x$$

$$q(x) = 40 \cdot 4^x$$

$$r(x) = 100 \cdot 4^x$$

- Use graphing technology to graph each function and to update your choice of accounts, if needed.
- Explain how changing the value of  $a$  changes the graph.

**Are you ready for more?**

As before, consider bank accounts whose balances are given by the following functions:

$$f(x) = 10 \cdot 3^x$$

$$g(x) = 3^{x+2}$$

$$h(x) = \frac{1}{2} \cdot 3^{x+3}$$

Which function would you choose? Does your choice depend on  $x$ ?

## 12.3

## Graphs Representing Exponential Decay

$$m(x) = 200 \cdot \left(\frac{1}{4}\right)^x$$

$$n(x) = 200 \cdot \left(\frac{1}{2}\right)^x$$

$$p(x) = 200 \cdot \left(\frac{3}{4}\right)^x$$

$$q(x) = 200 \cdot \left(\frac{7}{8}\right)^x$$



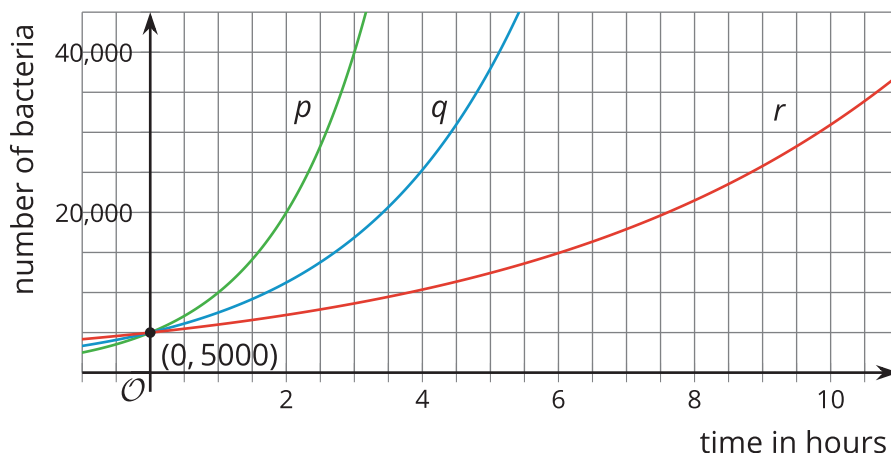
- Match each equation with a graph. Be prepared to explain your reasoning.
- Functions  $f$  and  $g$  are defined by these two equations:  $f(x) = 1,000 \cdot \left(\frac{1}{10}\right)^x$  and  $g(x) = 1,000 \cdot \left(\frac{9}{10}\right)^x$ .
  - Which function is decaying more quickly? Explain your reasoning.
  - Use graphing technology to verify your response.

## Lesson 12 Summary

An exponential function can give us information about a graph that represents it.

For example, suppose that function  $q$  represents a bacteria population  $t$  hours after it is first measured, and  $q(t) = 5,000 \cdot (1.5)^t$ . The number 5,000 is the bacteria population measured, when  $t$  is 0. The number 1.5 indicates that the bacteria population increases by a factor of 1.5 each hour.

A graph can help us see how the starting population (5,000) and growth factor (1.5) influence the population. Suppose functions  $p$  and  $r$  represent two other bacteria populations and are given by  $p(t) = 5,000 \cdot 2^t$  and  $r(t) = 5,000 \cdot (1.2)^t$ . Here are the graphs of  $p$ ,  $q$ , and  $r$ .



All three graphs start at 5,000, but the graph of  $r$  grows more slowly than does the graph of  $q$ , while the graph of  $p$  grows more quickly. This makes sense because a population that doubles every hour is growing more quickly than one that increases by a factor of 1.5 each hour, and both grow more quickly than a population that increases by a factor of 1.2 each hour.