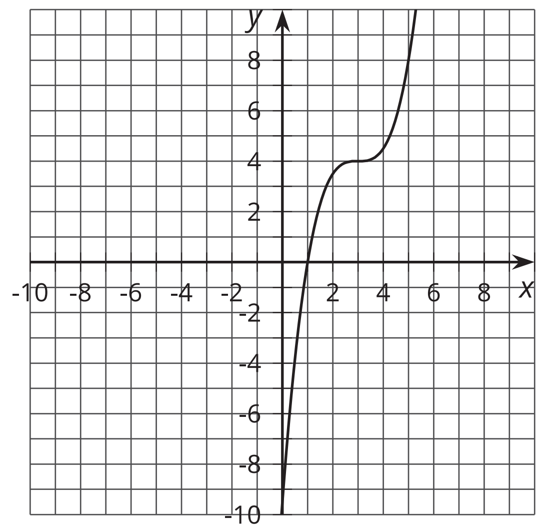
Unit 6, Lesson 12

# Transformation Effects

Let's transform some equations of functions.

## 12.1 Function Rewind

Here is a graph of $y = \frac{1}{2}(x-3)^3 + 4$. What transformations could you do to transform this function back to the original function $f(x) = x^3$?




## 12.2 Card Sort: Function Moves

Your teacher will give you a set of cards that show equations and graphs.

1. Sort the cards into categories of your choosing. Be prepared to describe your categories. Pause here for a class discussion.
2. Sort the cards into new categories in a different way. Be prepared to describe your new categories.

## 12.3 Same Transformation, Different Function

An original function is called $f$. The function $g$ is transformed from $f$ using the following transformations, in this order:

- Translate left 1
- Stretch horizontally by a factor of 3
- Reflect over the $x$-axis
- Stretch vertically by a factor of 6

1. Write an equation for $g(x)$ if $f(x)$ is:
   a. $x^2$
   b. $x^3$
   c. $e^x$
   d. $\sqrt{x}$
2. How could you write an equation for $g(x)$ in terms of an unknown function $f(x)$?

### Are you ready for more?

Consider the functions $f(x) = x^2$ and $g(x) = 3x^2$.

1. What is the average rate of change from $x = 0$ to $x = 1$ for each function?
2. What is the average rate of change from $x = 1$ to $x = 3$ for each function?
3. If the average rate of change between 2 points on $f(x)$ is -5, what would you expect the average rate of change for the same points on $g(x)$ to be? Explain your thinking.

4. Without calculating, how would the average rates of change compare for the function $h(x) = \left(\frac{1}{3}x\right)^2$ for the same values? Explain your reasoning.

## Lesson 12 Summary

We can examine the equation of a function to determine what transformations have taken it from an original function. Here is an example:

$g(x) = 2(3x + 4)^2 - 6$

If the original function is $f(x) = x^2$, then we can identify the transformations from $f(x)$ to $g(x)$:

- Shift left 4
- Horizontal stretch by a factor of $\frac{1}{3}$, which compresses the graph
- Vertical stretch by a factor of 2
- Shift down 6

We can see that the stretch by a factor of $\frac{1}{3}$ and the shift by 4 must be horizontal transformations because they are grouped with the input of the function. The stretch by a factor of 2 and shift by 6 must be vertical transformations because they are affecting the output of the function.

Now let's consider a new function:

$g(x) = 2 \cdot \frac{1}{3x+4} - 6$

If the original function is $f(x) = \frac{1}{x}$, then we can identify the transformations from $f(x)$ to $g(x)$ for this pair of functions also:

- Shift left 4
- Horizontal stretch by a factor of $\frac{1}{3}$
- Vertical stretch by a factor of 2
- Shift down 6

These are the exact same transformations as the first pair, even though the functions are very different! In fact, we can identify the transformations for an unknown original function using an equation in the same way. If $g(x)$ is a transformation of an unknown function $f(x)$ that has been shifted left 4, horizontally stretched by a factor of $\frac{1}{3}$, vertically stretched by a factor of 2, and shifted down 6, we can write a general equation for $g(x)$:

$g(x) = 2f(3x + 4) - 6$