Unit 1, Lesson 3

There's an Inequality Describing Any Triangle's Sides

Let's formalize our conjecture about sides in a triangle.

3.1 True Statements

Fill in each blank with a value or symbol to make the statement true.

1. a. $4 < 10 - \_\_\_\_$

   b. $4 = 10 - \_\_\_\_$

   c. $4 \geq 10 - \_\_\_\_$

2. a. $2 + \_\_\_\_ < 5$

   b. $2 + \_\_\_\_ = 5$

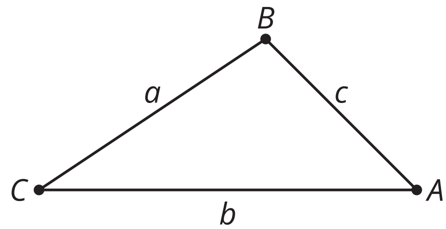
   c. $2 + \_\_\_\_ > 5$

3. a. $8 \_\_\_\_ 10 - 2$

   b. $7.9 \_\_\_\_ 10 - 2$

   c. $8.1 \_\_\_\_ 10 - 2$

3.2 Make It Mathy

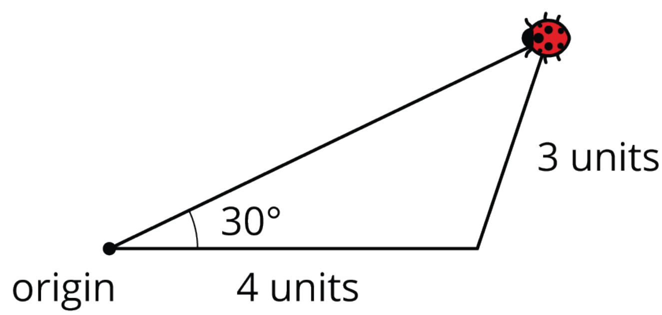


1. Rewrite the conjecture "In a triangle, any one side's length must be less than the sum of the other two sides' lengths" as an inequality that describes this triangle.

2. Write a convincing argument for why that conjecture must be true.

3.3 Where's the Ladybug?

A ladybug approximately the size of 1 unit travels 4 units east. She then travels a distance of 3 units in an unknown direction. Her total travel time is 15 seconds. Using a protractor, an entomologist measures that the ladybug was 30 degrees north of her original walking direction. The diagram shows one possibility for the ladybug's path and final location.



1. Is there a different final location that also matches the given information? If so, draw the ladybug there and use a different color to trace that path.
2. How many triangles can you draw given this information? Are these triangles congruent?

Are you ready for more?

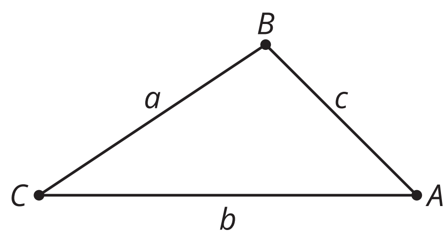
Suppose we have three points A, B, and C, where point A lies directly between points B and C. We are not considering ABC to be a triangle since it has no area, but it still makes sense to consider the distances $AB, AC,$ and BC. Mathematicians sometimes refer to ABC as a *degenerate triangle*.

1. Why might it make sense to consider such a triangle in the context of the ladybug?

2. How could we modify our statement of the Triangle Inequality Theorem to include degenerate triangles?

Lesson 3 Summary

The Triangle Inequality Theorem states that, for any triangle ABC, the length of any side must be between the difference and sum of the other two sides' lengths.



Given two sides of a triangle, you can figure out possibilities for the third side. For example, if two sides of the triangle are 4 centimeters and 6 centimeters, the third side must be between the difference $4 - 6 = 2$ and the sum $4 + 6 = 10$. Now you can quickly see that numbers like 3, 4.5, and 7.921 are options for the third side length, but 1, 2, 10 and 100 are not.

When given three lengths and asked to decide if those lengths form the sides of a triangle, you can pick two lengths and do mental math to find the sum and difference. If the third length is between those two values, then the three lengths can form a triangle. If you are given 5, 10 and 15, you might pick 5 and 10. In order for these three lengths to form a triangle, 15 must be between $10 - 5 = 5$ and $10 + 5 = 15$. Since 15 is not between 5 and 15, these lengths would not form a triangle.

Adding an angle into the given information narrows down the possibilities for a triangle's side lengths. When we are given two sides of a triangle and the angle that is not between those two sides, we can form up to two unique triangles. Sometimes we are given enough information, such as three side lengths, two side lengths and the angle between them, or two angles and one side length, to form a unique triangle.

