Unit 2, Lesson 10

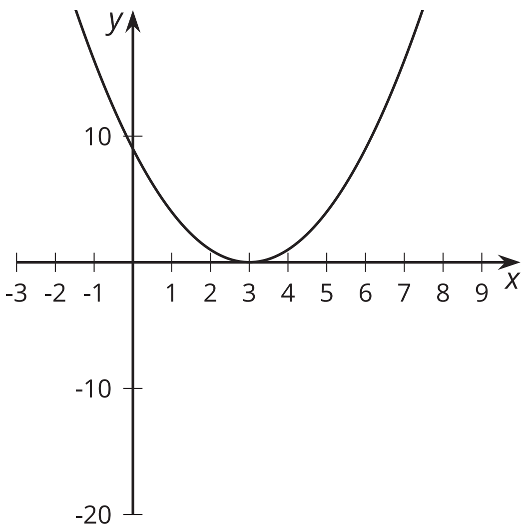
Multiplicity

Let’s sketch some polynomial functions.

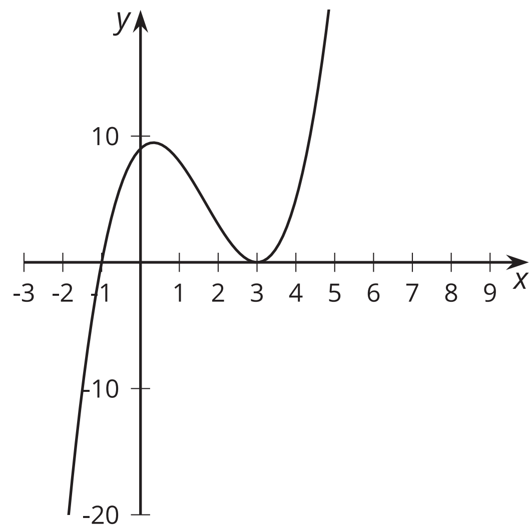
10.1 Notice and Wonder: Duplicate Factors

What do you notice? What do you wonder?

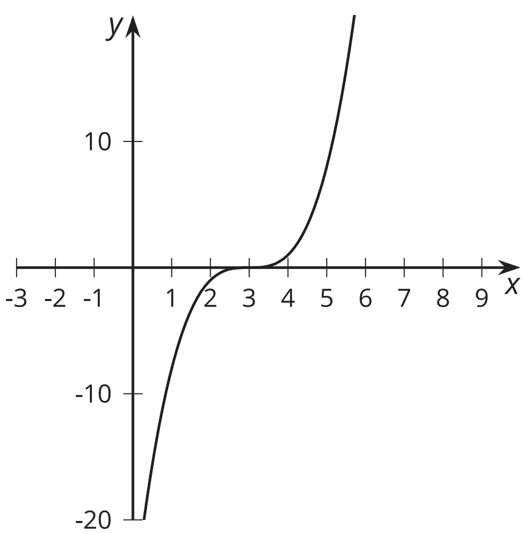
$y=(x-3)^2$



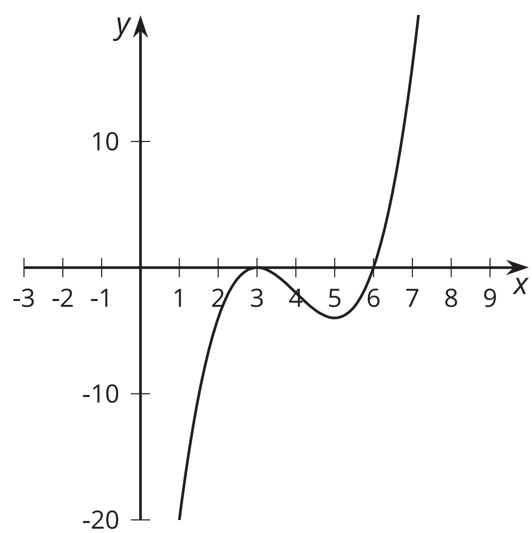
$y=(x+1)(x-3)^2$



$y=(x-3)^3$



$y=(x-6)(x-3)^2$



10.2 Sketching Polynomials

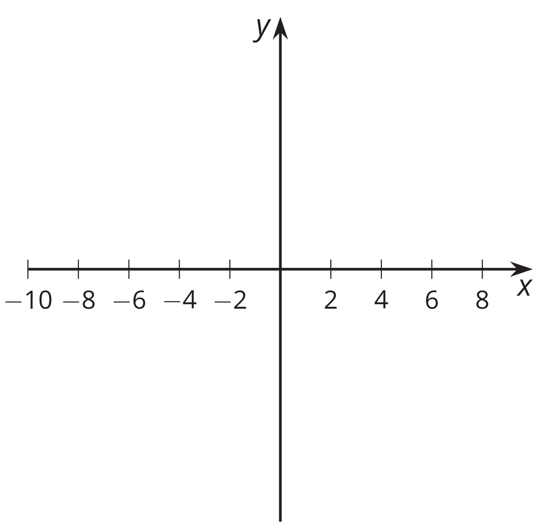
1. For polynomials A–F:
    a. Write the degree, all zeros, and complete the sentence about the end behavior.
    b. Sketch a possible graph.
    c. Check your sketch using graphing technology.

    Pause here for your teacher to check your work.

2. Create your own polynomial for your partner to figure out.
    a. Create a polynomial with degree greater than 2 and less than 8, and write the equation in the space given.
    b. Trade papers with a partner, then fill out the information about their polynomial and complete a sketch.
    c. Trade papers back. Check your partner's sketch using graphing technology.

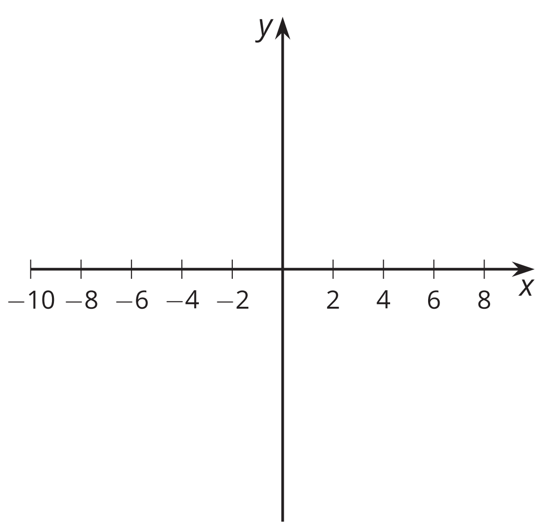
$A(x) = (x+2)(x-2)(x-8)$

Degree: Zeros:
End behavior: As x gets larger and larger in the negative direction,



$B(x) = \text{-}(x+2)(x-2)^2$

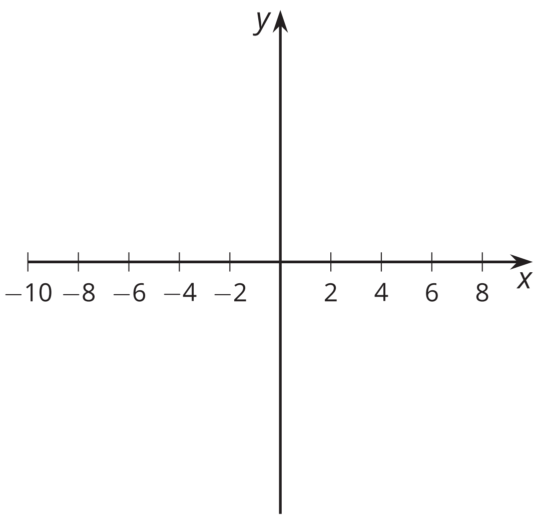
Degree: Zeros:
End behavior: As x gets larger and larger in the negative direction,



$C(x) = (x+6)(x+2)^2$

Degree: Zeros:

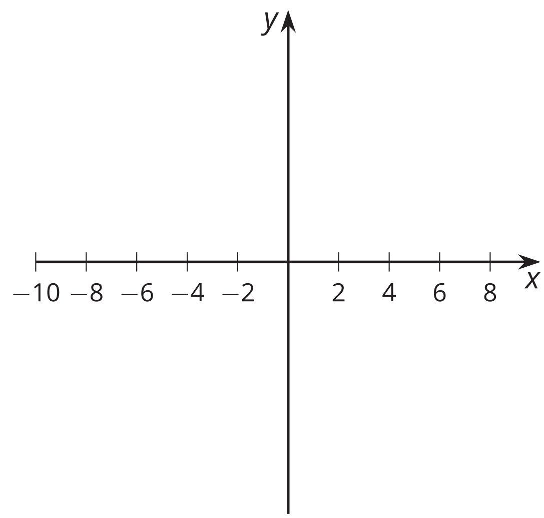
End behavior: As x gets larger and larger in the negative direction,



$D(x) = -(x+6)^2(x+2)$

Degree: Zeros:

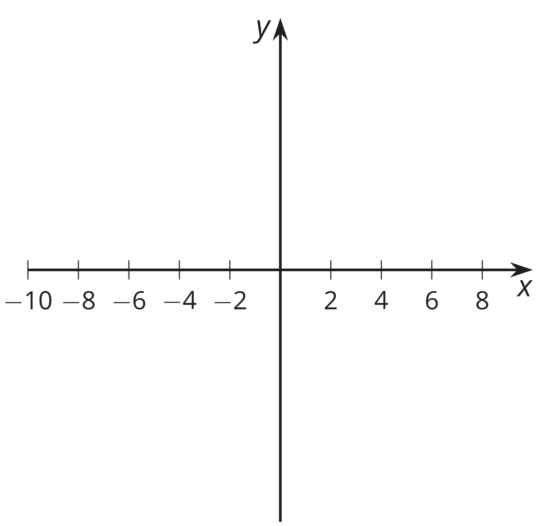
End behavior: As x gets larger and larger in the negative direction,



$E(x) = (x+4)(x-2)^3$

Degree: Zeros:

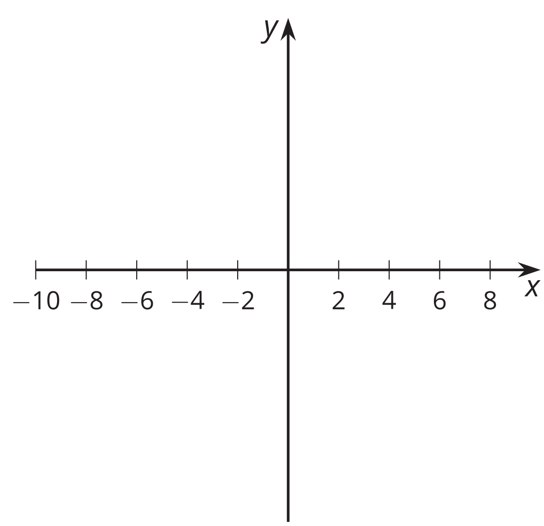
End behavior: As x gets larger and larger in the negative direction,



$F(x) = x^3(x+4)(x-3)^2$

Degree: Zeros:

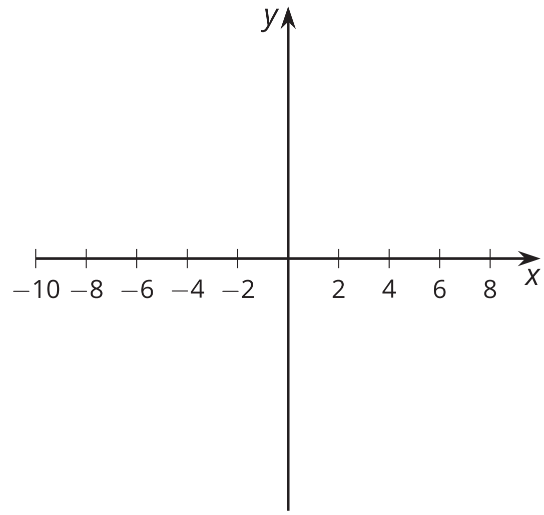
End behavior: As x gets larger and larger in the negative direction,



Your polynomial:

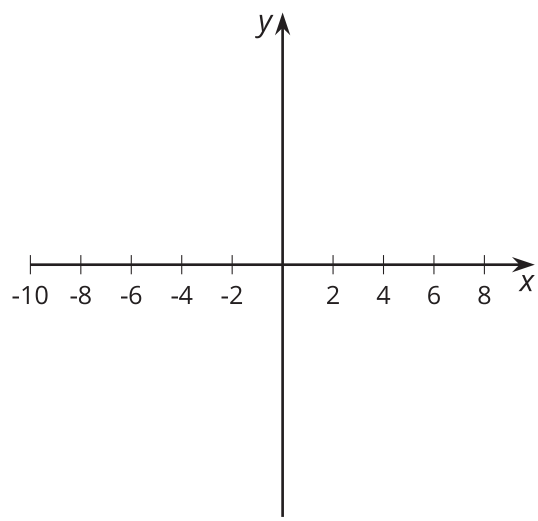
Degree: Zeros:

End behavior: As x gets larger and larger in the negative direction,



10.3 Using Knowledge of Zeros

1. Sketch a graph for a polynomial function $y = f(x)$ that has 3 different zeros and $f(x) \geq 0$ for all values of x.



2. What is the smallest degree the polynomial could have?

3. What is a possible equation for the polynomial? Use graphing technology to see if your equation matches your sketch.

Are you ready for more?

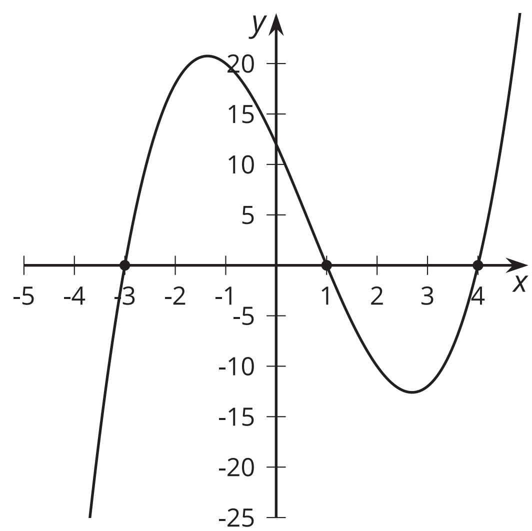
What is a possible equation of a polynomial function that has degree 5 but whose graph has exactly three horizontal intercepts and crosses the x-axis at all three intercepts? Explain why it is not possible to have a polynomial function that has degree 4 with this property.

Lesson 10 Summary

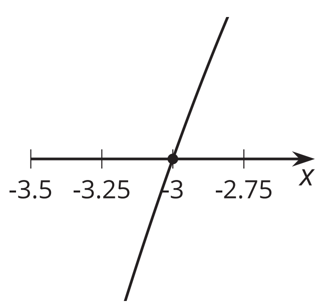
We can combine what we know about factors, degree, end behavior, the sign of the leading coefficient, and multiplicity to sketch polynomials written in factored form. **Multiplicity**, or the power to which a factor occurs in the factored form of a polynomial, tells us the number of times that factor is repeated and affects the shape of the graph near the location of each zero on the horizontal axis.

For example, $y = (x+3)(x-1)(x-4)$ has three factors with no duplicates. We say that each factor, $(x+3)$, $(x-1)$, and $(x-4)$, has a multiplicity of 1. This results in a graph that looks a bit like a linear function near $x = \text{-}3$, $x = 1$, and $x = 4$ when we zoom in on each of those places.

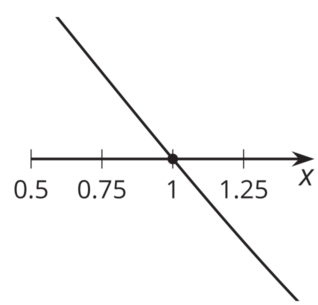
$y = (x+3)(x-1)(x-4)$



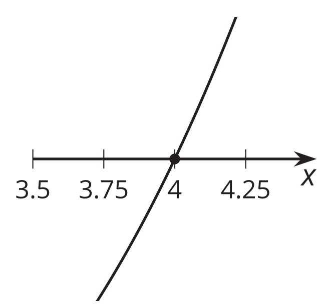
near $x = \text{-}3$



near $x = 1$

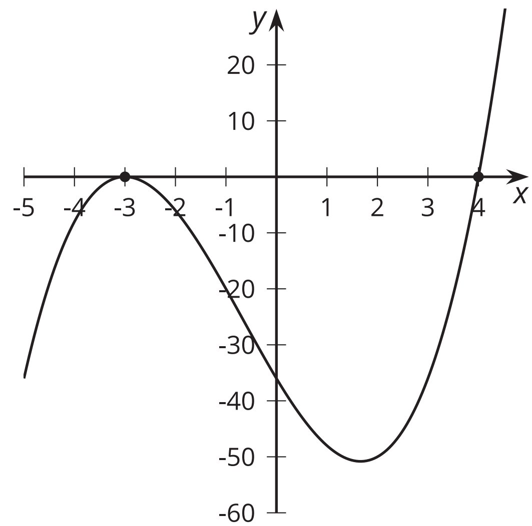


near $x = 4$

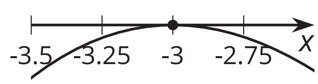


For $y = (x+3)^2(x-4)$, there are still three factors, but two of them are $(x+3)$. This results in a graph that looks a bit like a quadratic near $x =$ -3 and a bit like a linear function near $x = 4$. We say that the factor $(x+3)$ has a multiplicity of 2 while the factor $(x-4)$ has a multiplicity of 1.

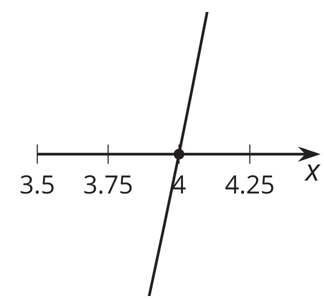
$y = (x+3)^2(x-4)$



near $x =$ -3



near $x = 4$



Now consider what the graph of $y = (x+3)(x-4)^3$ would look like. The factors help us identify that the function has zeros at -3 and 4. We also know that since $(x+3)$ has a multiplicity of 1 and $(x-4)$ has a multiplicity of 3, the graph looks a bit like a linear polynomial crossing the x-axis at -3 and a bit like a cubic polynomial crossing the x-axis at 4. Since this is a 4th-degree polynomial with a positive leading coefficient, we know that as x gets larger and larger in either the negative or positive direction, y gets larger and larger in the positive direction.

$y = (x+3)(x-4)^3$